Supersaturated Steam

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Supersaturation, or the failure of steam to condense when the saturated condition is reached in an expansion, has long been a matter of interest to engineers. This paper presents the account of an investigation of this phenomenon in which the principal objectives were the location of the Wilson line, which indicates the condition at which condensation actually occurs, and the measurement of the size of the drops which are formed when flowing steam condenses. Examination of the flow of low pressure steam through an illuminated nozzle fitted with a glass top revealed that the condensation process could be observed and that the pressure of the steam at the condensation point could be measured with the aid of a search tube. It was found that in a simple convergent-divergent nozzle condensation did not occur until the steam had reached the condition approximately represented by the 3.5 per cent moisture line on the Mollier chart. The droplets which formed at that condition were extremely minute, their radii being of the order of magnitude of 6.2 $\times$ 10^{-8} cm, and there was no evidence of growth during their passage through the nozzle. Additional experiments with nozzles of other designs led to the discovery that under certain conditions condensation could take place when the steam had reached the 2 per cent moisture condition, and that the droplets so formed grew rapidly from about 10.0 $\times$ 10^{-8} cm to about 6.0 $\times$ 10^{-5} cm in radius. It was concluded that supersaturation invariably occurred in the condensation of flowing steam, and that the superheated steam formula should be used to estimate the flow of saturated steam through nozzles.

An accurate knowledge of the weight of steam which may be expected to flow through a given nozzle area under varying conditions is essential in the design of steam turbines and other equipment. The flow of superheated steam can be estimated with a high degree of accuracy by the use of the Saint-Venant formula. Applying the usual formula to the flow of saturated or wet steam, however, with the assumption that condensation begins as soon as the saturation condition is passed in an expansion, results in a theoretical flow which may be smaller than the actual. This excess, at first attributed to experimental error, has been so conclusively demonstrated by the work of able experimenters that the correctness of the conventional theory of condensation is questionable.

The theory of supersaturation was proposed to explain the phenomena involved in the flow of saturated or wet steam. It was suggested that steam in a rapid expansion from a dry or slightly superheated condition might not begin to condense when the saturated condition was reached, but might continue to expand as in the superheated region, thus becoming supersaturated. Such a theory explains the excess flow encountered in the expansion of saturated steam through nozzles.

Flow of a fluid through a nozzle depends upon the product of its density and velocity at any given cross-section. The conventional theory of condensation requires that steam begins to condense when the saturated condition is passed in an expansion. The condensed portion of the steam gives up its latent heat, causing a decrease in the density of the surrounding medium. If the condensation should fail to occur, the latent heat would be retained with the result that, for a given expansion, the density of supersaturated steam would be greater than that of wet steam. The velocity of the supersaturated steam would be less than that of the wet steam because the isentropic heat drop is less for a supersaturated than for an equilibrium expansion. The product of the velocity and the density, however, is greater for the supersaturated than for the wet steam and hence the weight flowing through a given area is greater.

The actual mechanism of condensation in flowing steam, which is by no means clearly understood, involves the formation of water droplets. The nature of formation and the size of these droplets is of great importance in determining their behavior, particularly in respect to the erosion of the low pressure blades in steam turbines.

Need for further data on supersaturation in actual steam flow and the lack of information on condensation nuclei and drop size led to a two-year study in the laboratories of the Johns Hopkins University of the flow through nozzles of relatively low-pressure steam. The original object of this research was to investigate the condensation of steam in an effort to learn how and when it occurs and to discover the conditions under which supersaturation actually exists. It was desired to locate by experimental means the Wilson line which, on the Mollier chart, represents the condition at which condensation actually occurs in expanding steam at the termination of the supersaturated state.

In considering methods of attack, it appeared that Mellanby and Kerr (25)[2] had exhausted the possibilities of the weighed-flow analysis. The optical method used by Stodola (29) to study flow in nozzles and by Thomas (31) to examine moisture in steam did not appear to have been utilized to the fullest possible extent, and the possibility that drop sizes might be measured by optical means made this method more attractive. It was believed that, if the condensation point could be seen, the pressure at that point could be measured with a search tube and thus some positive knowledge of condensation conditions would be obtained. From this the supersaturation could be determined.

2 Historical

Supersaturation has been studied by many investigators. Aitken (1) concluded that all condensation occurred on dust particles as nuclei and he devised a dust counter which utilized this principle.

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[2] Numbers in parenthesis refer to the bibliography, Appendix 3, at the end of this paper.

Von Helmholtz (18) called attention to the supersaturation of steam issuing from an orifice into the open air. Observing that electrification of the jet greatly increased the number of droplets, he came to the conclusion that ions usually served as nuclei for condensation. The extension of Lord Kelvin's formula for the equilibrium pressure over curved surfaces is due to von Helmholtz.

G. T. R. Wilson (34–37), whose name is given to the line on the Mollier chart which marks the limit of supersaturation, investigated the nature of this phenomenon. By expanding in a glass chamber a sample of air or other gas saturated with moisture, he was able to measure the pressure at which condensation began. From this he deduced the supersaturation ratio by dividing the pressure at which condensation started by the saturation pressure corresponding to the temperature of the supersaturated steam. At —16 C, Wilson found a supersaturation ratio of 7.9, corresponding to a drop radius of 6.4×10^{-8} cm.

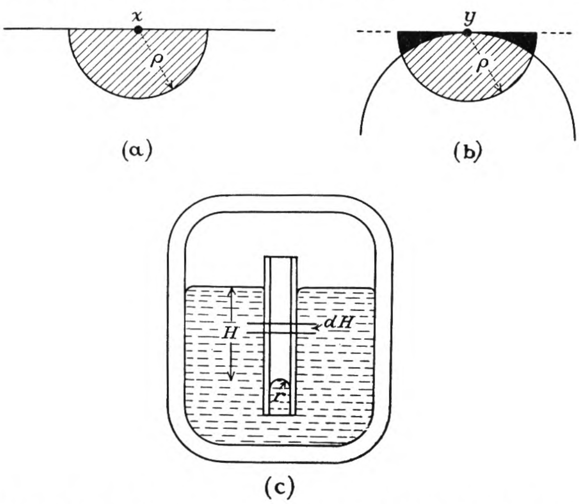

Fig. 1 Vapor Pressure Above Plane and Curved Surfaces

J. J. Thomson (33) studied with great care the effect of ionization upon supersaturation and developed the theory underlying condensation upon ions. In general, his findings support those of Wilson.

Carl Barus (4–8) likewise obtained results which substantiated Wilson's. He was particularly interested in the problem of supersaturation as applied to meteorological phenomena.

C. F. Powell (27), at Wilson's suggestion, investigated with improved apparatus the supersaturation of saturated air at four temperatures. His results will be discussed later.

A. Stodola (29), by his work with illuminated glass nozzles, found positive evidence of the existence of supersaturation in actual nozzle flow. With characteristic thoroughness he repeated the work of earlier experimenters and then developed the method of attack which is used in this investigation. He has reported one measurement made in this manner in his book, "Steam and Gas Turbines" which contains a complete presentation of the theory of supersaturation.

Mellanby and Kerr (25) weighed and analyzed the flow of saturated steam through nozzles and concluded that, to account for their experimental results, supersaturation must exist.

Using the weighed flow method, Kearton (22) studied the flow of mercury vapor through nozzles and showed that supersaturation occurred during that process. The limitations of this method prevented him from placing a definite value upon the supersaturation to be expected with mercury.

Supersaturation has also been studied from purely theoretical considerations. Maxwell (24), discussing the Thomson isothermal, first suggested the possibility of a supersaturated state for steam.

Sir William Thomson, Lord Kelvin (32) gave a theoretical foundation to supersaturation when he derived the relation between the vapor pressures above a curved and a plane liquid surface. His equation is the basis of all supersaturation calculations.

H. L. Callendar (9–11), studying the so-called missing quantity in reciprocating steam engines, concluded that supersaturation could account for at least a portion of that quantity. He discussed supersaturation at length in several publications and in his book, "The Properties of Steam."

H. M. Martin (23) brought the subject of supersaturation to the attention of engineers by his paper, "A New Theory of the Steam Turbine." He assumed that condensation always results in droplets of the same size. On this basis he applied Callendar's value of the droplet radius to the von Helmholtz equation, making allowance for the variation of surface tension with temperature and estimated the supersaturation ratio which might be expected at any temperature. From these data, using the Callendar equations of state, he calculated the properties of steam for the conditions under which condensation should occur. His results, plotted on the Mollier chart, form the line to which he gave Wilson's name.

Goodenough (16) presented a thorough discussion of the theory of supersaturation and its effects and then, doubting the validity of the theory, proceeded to show that the same effect of increasing the flow over the calculated value could be produced by the presence in the steam of small water particles. His article is of particular interest for its exposition of the effect of water droplets upon velocity coefficients of nozzles.

3 Theory of Supersaturation

Supersaturation is based upon the fact that the vapor pressure of a liquid at a given temperature is greater above a curved than above a plane surface. This can be understood by referring to Fig. 1 where (*a*) shows a molecule, x, on a plane liquid surface, which is attracted by all of the molecules of the liquid within the hemisphere of radius ρ, the distance over which the molecular attraction may be assumed to act, and (*b*) shows a molecule, y, on a curved surface, which is attracted by fewer molecules than x because those molecules in the solid area are no longer present. At the same temperature, then, molecule y will be tied less securely to the surface than will x, and conversely a greater pressure in the surrounding atmosphere will be required to force y to remain on the surface. In other words, the vapor pressure in case (*b*) is greater than that in (*a*). Thus a water droplet, if sufficiently small, will evaporate if placed in an atmosphere of saturated steam. If, however, the vapor is supersaturated or at a pressure greater than that corresponding to the temperature, equilibrium can exist between the vapor and the droplet when the pressure of the vapor is equal to the vapor pressure of the droplet. The conditions necessary for equilibrium between vapor and droplets were investigated by Lord Kelvin (32) and a formula to express these conditions was derived by von Helmholtz (18).

Lord Kelvin's equation evaluates the change in vapor pressure of a liquid caused by a change in the curvature of its surface. Fig. 1(*c*) shows the assumed conditions. A capillary tube of radius r is inserted into a liquid of which γ is the surface tension in lb per ft. The whole is enclosed in a vessel so that only the liquid and its vapor, both at temperature T, are present. If the liquid does not wet the tube, it will be depressed due to capillary action to a level below that of the plane surface of the remaining liquid in the vessel. The surface of the liquid in the tube may be assumed to be a hemisphere of radius r. The distance of the meniscus below the level of the liquid is H, and P and p are the pressures in the liquid and the vapor, respectively, at the meniscus,

$$\pi r^2 P = \pi r^2 p + 2\pi r\gamma \qquad [1]$$

Rearranging and eliminating πr,

$$P - p = \frac{2\gamma}{r} \qquad [1a]$$

When the concavity is downward r is assumed to be positive. Due to the difference in elevation, H, the pressure of the vapor, p, at the curved surface in the tube must be greater than that at the plane surface. The vapor, however, must be in equilibrium with both the curved and the plane surfaces; otherwise perpetual motion would result. Therefore, denoting by P_o and p_o the pressure in the liquid and in the vapor, respectively, at the plane surface,

$$P_o = p_o \qquad [2]$$

If v is the specific volume of the vapor, the difference in pressure due to the difference in elevation will be

$$p - p_o = \frac{H}{v} \qquad [3]$$

Likewise, if V is the specific volume of the liquid, the difference in the liquid pressures will be

$$P - P_o = \frac{H}{V} \qquad [4]$$

We wish now to solve for $p - p_o$, the change in vapor pressure caused by the curvature of the surface.
Combining [3] and [4] and eliminating H,

$$(p - p_o) = (P - P_o)\,\frac{V}{v} \qquad [5]$$

Inverting [3] and [4] and subtracting,

$$\frac{(v - V)}{H} = \frac{1}{p - p_o} - \frac{1}{P - P_o} \qquad [6]$$

and recalling from [2] that $P_o = p_o$,

$$\frac{v - V}{H} = \frac{P - P_o - p + p_o}{(p - p_o)\,(P - P_o)} = \frac{P - p}{(p - p_o)\,(P - P_o)} \qquad [6a]$$

Since $(P - P_o) = \dfrac{H}{V}$,

$$\frac{v - V}{V} = \frac{P - p}{p - p_o} \qquad [7]$$

Substituting [1a] in [7], we have the desired relation:

$$p - p_o = \frac{2V\gamma}{r(v - V)} \qquad [8]$$

Thus, for equilibrium to exist, the pressure of a vapor containing drops of radius r must exceed the vapor pressure of the drops by $\dfrac{2V\gamma}{r(v - V)}$, which may be very large if r is sufficiently small.

The equation first derived by von Helmholtz expresses the same relation in more convenient form. Referring again to Fig. 1(*c*) and Equation [2], we may define the pressure of the vapor at the plane surface, p_o, as the saturation pressure at temperature T, p_s. Rearranging Equation [1a] and subtracting p_s from each side of the equation, we have,

$$p - p_s + \frac{2\gamma}{r} = P - p_s = P - P_o \qquad [9]$$

since $p_s = p_o$ by definition and $p_o = P_o$
Equation [9] can now be written:

$$\frac{2\gamma}{r} + \int_{p_s}^{p} dp = \int_{P_o}^{P} dP \qquad [9a]$$

Consider an elementary section, dH, on Fig. 1(*c*). The increase of the liquid pressure $dP = \dfrac{dH}{V} = D\,dH$, where D, the density of the liquid, equals $\dfrac{1}{V}$. The increase of the pressure of the vapor, $dp = \dfrac{dH}{v} = D\,dH$, where d is the density of the vapor. Thus $dP = D\,dH$, and $dp = d\,dH$, so $dP = \dfrac{D}{d}\,dp$. Substituting in [9], we have:

$$\frac{2\gamma}{r} = \int_{p_s}^{p} \frac{D\,dp}{d} - \int_{p_s}^{p} dp = \int_{p_s}^{p} \left(\frac{D}{d} - 1\right) dp \qquad [10]$$

In general, d is very small compared to D, and $\dfrac{D}{d}$ is so large compared to 1 that without serious error we may omit the product $1 \times dp$. Likewise, over a small range, D may be taken as a constant, so we have:

$$\frac{2\gamma}{r} = D \int_{p_s}^{p} \frac{dp}{d} \qquad [11]$$

Assuming that the perfect gas relationships are applicable to this case,

$$pv = RT \text{ and } \frac{1}{d} = v = \frac{RT}{p}$$

Equation [11] can now be rewritten and integrated:

$$\frac{2\gamma}{r} = RTD \int_{p_s}^{p} \frac{dp}{p} = RTD \log_e\left(\frac{p}{p_s}\right) \qquad [12]$$

which we may express as:

$$\log_e\left(\frac{p}{p_s}\right) = \frac{2\gamma}{RTDr} = \log_e S \qquad [13]$$

where p denotes the pressure in equilibrium with the drop of radius r at temperature T, and p_s is the saturation pressure at that temperature. This is the fundamental supersaturation equation of von Helmholtz.

The ratio $\dfrac{p}{p_s}$, denoted by the symbol S, is known as the supersaturation ratio. Equation [13] gives the relation which must exist between the actual pressure and the saturation pressure at the existing temperature if a drop of radius r is to be in equilibrium with the vapor about it. If this ratio is lowered, the droplet will evaporate. If it is raised, the droplet will grow.

It will be seen from Equation [13] that the logarithm of the supersaturation ratio varies directly with the surface tension and inversely with the absolute temperature and the radius of the drops. Surface tension decreases with increasing temperature and finally vanishes at the critical condition. Thus there can be no supersaturation at the critical condition. Conversely, surface tension increases with decreasing temperature and the supersaturation ratio for a given size of drop increases as the temperature is lowered.

Equation [13] also indicates the need for a nucleus upon which condensation can occur. If r is made very small, S must be very large and finally, in the limiting case, when $r = 0$, $S = \infty$. Thus in order for a pure vapor to condense, there must be some nucleus of finite radius.

The search for such a nucleus motivated early workers in this field. Aitken at first concluded that dust particles always served as nuclei, but he rejected this conclusion upon finding that condensation occurred with filtered, dust-free vapor. Von Helmholtz believed that ions were the nuclei upon which condensation took place.

G. T. R. Wilson found three distinct types of nuclei. Working with air saturated with moisture, he found that a small expansion caused condensation upon the relatively large particles of dust which were present in the air. With dust-free air he discovered that at a higher expansion ratio condensation occurred upon ions. Wilson's "cloud chamber," well known to physicists, is based on this feature of the phenomenon. He was able to trace the path of ionizing rays by causing them to pass through an illuminated chamber and photographing their tracks, which are droplets of water condensed upon the ions formed by the ray.

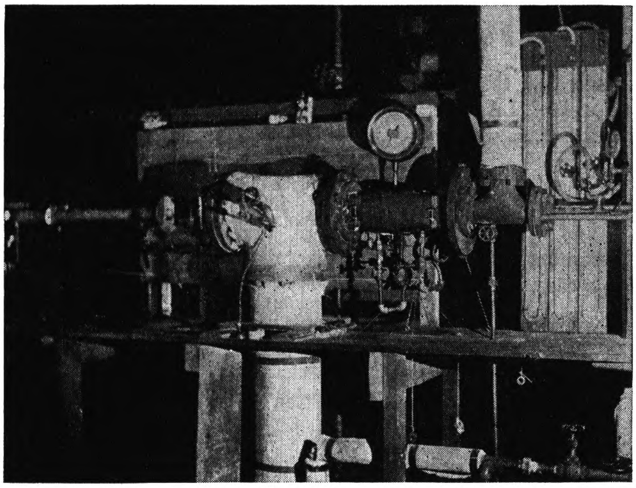

Fig. 2 Apparatus Used in Investigation

With air freed both of dust and of ions, Wilson found that a sufficiently high ratio of expansion would invariably cause a heavy condensation in the form of innumerable very small droplets. The nuclei responsible for this ultimate condensation, investigated at great length by Barus (6–8), were seemingly inexhaustible, and appeared to form an essential part of the water vapor, for they could not be removed by any process. The conclusion reached by both Barus and Wilson was that these nuclei are associated or agglomerated molecules.

Callendar (9–12), assuming that the time interval in the flow of steam through nozzles is too small to allow condensation to occur, used Wilson's value of 7.9 for the supersaturation ratio at 300 C, abs, in the von Helmholtz equation, and obtained 5.0×10^{-8} cm as the radius of the droplets formed when condensation takes place. The Wilson line, in his opinion, approximately coincided with the 3 per cent moisture line on the Mollier chart. Martin's original Wilson line lay between the 3 and 4 per cent moisture lines.

Powell's calculations resulted in a droplet radius of 6.4×10^{-8} cm, and his version of the Wilson line falls along the 2 per cent moisture line.

While the existence of supersaturation has been demonstrated by the work of the authorities mentioned above, a supersaturation limit, or Wilson line, based on theory alone is not acceptable. There are so many uncertain variables in the von Helmholtz equation and so many assumptions in its derivation that the validity of results based upon it is open to question. An experimental investigation of the condensation conditions is therefore necessary to determine the actual supersaturation limit, and the following sections record such an investigation.

4 Description of the Apparatus

The apparatus, Fig. 2, was designed to provide a nozzle with a transparent side through which the expanding steam could be observed with the aid of an intense beam of light passing axially through the nozzle. Observation at right angles to the illuminating beam was essential because, in this way otherwise invisible details could be seen as in the ultra-microscope. Referring to Fig. 3, steam entered through the main valve A and passed through a 2-in. line into the superheater, B, consisting of a 6-in.

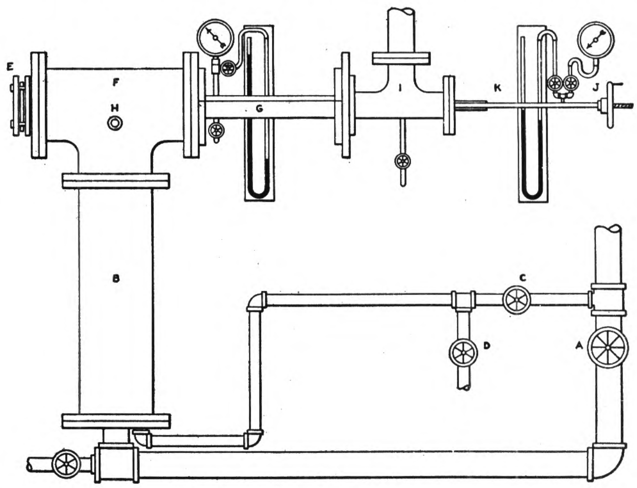

Fig. 3 Diagram of Apparatus

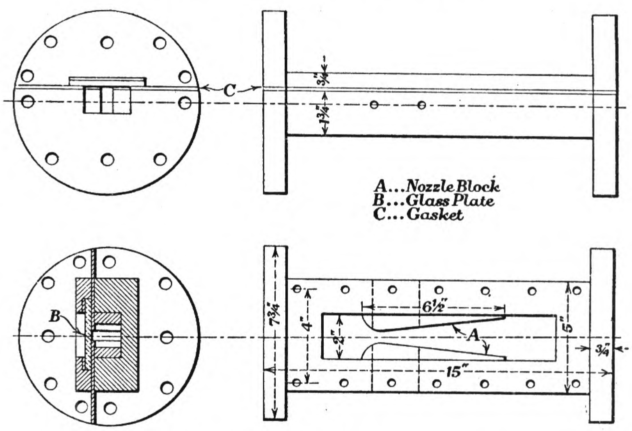

Fig. 4 Details of Nozzle Design

steel pipe within which was placed a copper coil supplied with high-pressure steam through valve C. Evaporation of the entrained moisture in the entering steam was the principal function of the superheater, but it could also be used as a desuperheater by closing valve C and the drain valve, and opening valve D, admitting to the coil cold water which was discharged to the sewer.

Steam passed from B into the tee, F, which had at one end a glass port, E, and at the other the nozzle assembly, G, shown in detail in Fig. 4. In the side of F (Fig. 3) was inserted a standard thermometer well which was provided with a mercury-in-glass thermometer to determine the temperature of the incoming steam.

The nozzle, rectangular in cross-section, was formed by two polished brass blocks bolted to the sides of a cast-iron channel. The glass plate which constituted the top of the nozzle was clamped tightly to the channel, with rubber gaskets above and below to prevent leakage. Fig. 5 shows the dimensions of the several nozzles used in the experiments.

The inlet pressure was measured by a Bourdon gage or a mercury manometer. The pressure connection, a $^1/_{32}$-in. hole, was located at the center of the bottom of the channel, 1 in. above the throat of the nozzle. The steam from the nozzle passed through a second tee into a 2-in. line leading to a condenser in which a vacuum of 25 in. of mercury could be obtained. Valves in the discharge line permitted the back pressure to be raised to any desired value.

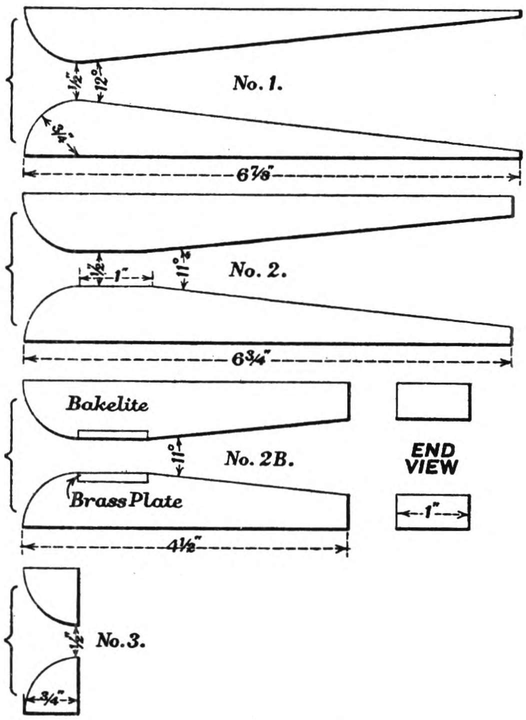

Fig. 5 Dimensions of Nozzle Blocks

The static pressure of the steam at any point along the axis of the nozzle could be measured with the aid of a brass search tube, of 0.125 in. inside diameter and 0.189 in. outside diameter. The steam pressure was transmitted through six holes, $^1/_{32}$-in. in diameter, in the form of a piezometer ring, located about 14 in. from the end of the tube. The search tube was soldered to a larger tube which passed through a stuffing box and was connected by means of flexible rubber tubing to a Bourdon gage and a mercury manometer. A hand-wheel and screw permitted the search tube to be moved along the axis of the nozzle, and the location of the pressure measuring holes could be found by a scale on the frame. To prevent vibration, the steam end of the search tube was held in place by a guide at the high pressure end of the channel. The back pressure was measured by a mercury manometer connected to a $^1/_{32}$-in. hole in the bottom of the channel beyond the nozzle mouth.

Illumination was provided by a carbon arc, the light from which was concentrated by a pair of lenses and introduced through the port E (Fig. 3) along the axis of the nozzle. Usually a screen was used to keep the light from hitting the bottom of the nozzle, and a blue filter could be interposed between the arc and the nozzle if needed. When full illumination was desired, the search tube was removed and in its place was installed a glass port through which the light from a second arc entered. This light could be focussed with a concave mirror into a sharp beam to study one portion of the nozzle or into a broad ray to illuminate it completely.

The pressure gages, calibrated frequently during the course of the work, could be read to within 0.5 lb per sq in. The probable error in the absolute pressure measurements thus varied between 0.84 per cent for low pressures and 0.35 per cent for high pressures. The mercury manometers were made of glass tubing 0.25 in. in diameter, mounted on varnished wood. The scales were made of portions of graph paper fastened to the wood and varnished to minimize the effect of humidity. The height of the columns could be read with an accuracy of 0.05 in., the probable error varying from 0.5 to 0.1 per cent. Correction was made for the water which condensed in the tubes by allowing 13.6 in. of water per in. of mercury. The barometric readings, obtained in an adjacent laboratory located about 10 ft above the apparatus, were not corrected for this difference in elevation, nor for the changes in room temperature, for such variations are of a small order of magnitude and have no significant effect.

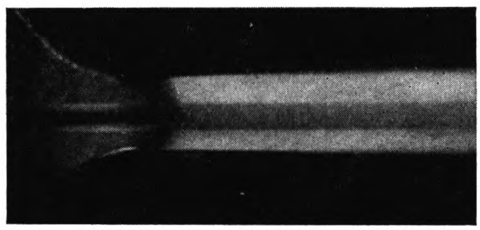

Fig. 6(*a*)

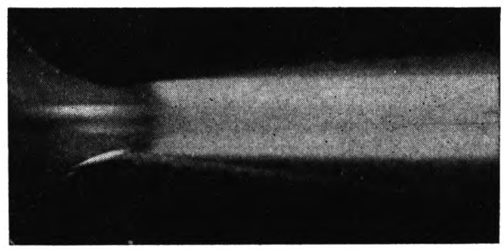

Fig. 6(*b*)

Fig. 6 Flow Through Nozzle No. 1

[*a* (upper)—Illumination confined to plane of search tube. *b* (lower)—Search tube in place. Entire cross-section illuminated. Initial conditions: 30 lb per sq in. gage, 300 F, 16 lb per sq in. abs back pressure.]

The thermometer was tested and found to be correct within 0.5 F. It could be read to within 0.5 F by means of a magnifying lens attached to it. The maximum probable error in the temperature measurements was thus 1.0 F in 300 or 0.33 per cent.

5 Method of Measuring the Supersaturation Ratio

Supersaturation cannot be measured directly but must be calculated from an observed condensation pressure. Attempts to measure the temperature of supersaturated steam will fail because such steam will condense on any surface and a thermometer placed in such an atmosphere will immediately become covered with a thin film of moisture. The temperature of saturated steam at the existing pressure will thus be obtained.

When the apparatus was put into operation it was found that the intense light from the arc enabled the condensation point to be observed. The entering steam, superheated by throttling from the boiler pressure, was quite transparent and the occasional drops of entrained moisture were easily seen. When condensation took place, however, the arc light was scattered by the great number of very small drops which formed in the steam, and a dense bluish mist was visible. As shown in Fig. 6(*a*), condensation occurred along a relatively sharp curved line which bore a marked resemblance to the constant pressure lines in nozzles shown by Stodola (29). There was a distinct and unmistakable change in appearance from the transparency of the dry supersaturated steam to the blue cloud which indicated the presence of minute water droplets. The incident light was scattered rather than reflected or refracted, and the only color to be seen was the blue which is characteristic of the light scattered by very small particles.

In determining the condensation pressure, the apparatus was allowed to run until it was thoroughly warm and the entering

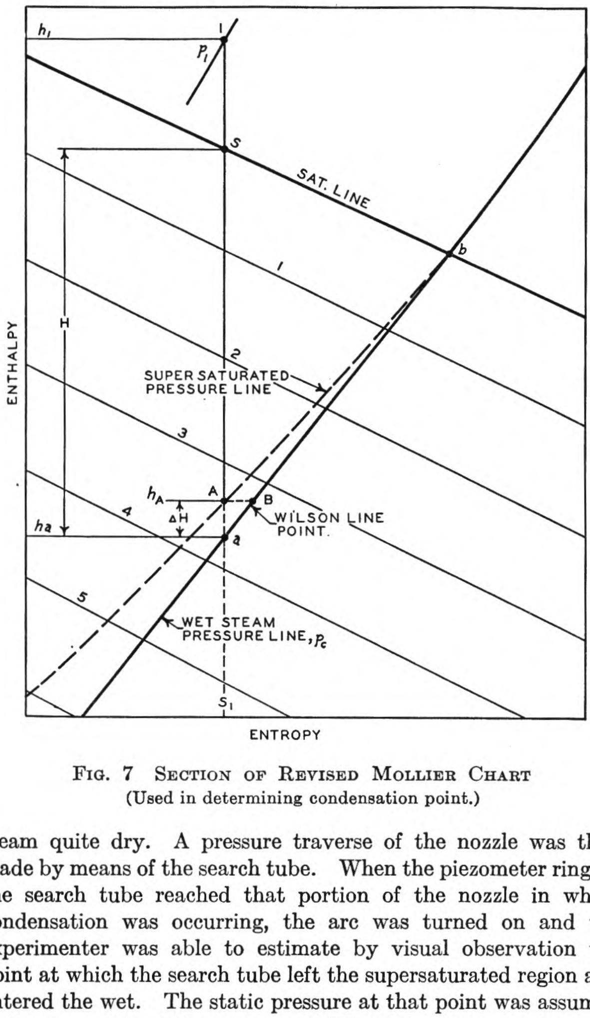

Fig. 7 Section of Revised Mollier Chart
(Used in determining condensation point.)

steam quite dry. A pressure traverse of the nozzle was then made by means of the search tube. When the piezometer ring on the search tube reached that portion of the nozzle in which condensation was occurring, the arc was turned on and the experimenter was able to estimate by visual observation the point at which the search tube left the supersaturated region and entered the wet. The static pressure at that point was assumed to be the condensation pressure. When the traverse was completed the condensation region was reexplored several times to check the pressure determination. The pressures thus obtained were plotted against nozzle length as in Fig. 8(*a*). The resulting curves resemble those shown by Stodola and by Mellanby and Kerr.

The method of determining the condition of the steam at the condensation point is based upon the assumption that the expansion through the throat of the nozzle is isentropic. This is not strictly true, for some losses are present, but those which occur before the throat is reached are small. It will be seen from Figs. 8(*a*) and 8(*b*) that the expansion to the throat of the nozzle used in this work was rapid and continuous. The fluctuations in pressure, which are responsible for a large part of the losses, occur after the condensation point has been reached. It is probable, therefore, that the losses to the condensation point do not at the most exceed 1 or 2 per cent of the isentropic enthalpy drop.

If isentropic expansion is assumed, the condition at the condensation point can be estimated from the Mollier chart by following the entropy line on which the initial-state point of the steam is located. The condensation condition is the intersection of that entropy line and the line representing the pressure at which condensation is observed. Since the conventional Mollier chart is based upon equilibrium conditions, the supersaturated condition cannot be represented correctly upon it. For this reason a revised Mollier chart was prepared in the manner described in Appendix 2. Fig. 7 is a portion of this chart plotted on a large scale and shows the method of locating the condensation point. Point 1 is the initial condition, determined by measuring the pressure and temperature of the incoming steam. Line *a-b* is the conventional wet steam line in thermal equilibrium for the pressure p_c at which condensation is observed and *A-b* is the supersaturated line for the same pressure.

Steam expanding isentropically from point 1, crosses the saturation line at point *S* but remains dry and supersaturated until it reaches a condition represented by point *A* on the supersaturated pressure line. At that point condensation takes place and thermal equilibrium is almost instantaneously restored. This action is represented by the constant enthalpy line *A-B*. The condition of the wet steam is thus represented not by point *A* or *a*, but by point *B*. If the expansion is not exactly isentropic, the friction and other losses will reheat the steam and raise slightly the enthalpy of point *B*.

It is evident that in a given expansion there is a distinct loss in availability because of this isenthalpic change from the supersaturated to the wet condition. If H is the isentropic enthalpy drop from the saturation line to the wet steam line *a-b*, ΔH is the portion rendered unavailable because of the increase in entropy attendant upon condensation. It will be shown later that condensation takes place between the 3 and the 4 per cent moisture lines and for this condition the loss in availability remains sensibly constant at about 2 Btu. Since the isentropic enthalpy drop from the saturation line to this condition is approximately 50 Btu, the loss in availability amounts to some 4 per cent of that enthalpy drop. This loss probably has an important bearing on the nozzle efficiencies' being lower with wet than with superheated steam. In turbine practise the supersaturation loss is usually covered by the assumption that stage efficiencies are lower in the region below the saturation line than in the superheated region. The usual allowance is 1 per cent decrease in efficiency for each per cent of moisture in the steam.

Martin's original definition (23) indicates that he intended the Wilson line to represent the condition of supersaturation at which condensation must occur in an expansion, that is, point *A* in Fig. 7. Since this requires a Mollier chart upon which the supersaturated pressure lines are present, it seems advisable to change this definition slightly, because the designer usually has at his disposal only the conventional chart. The Wilson line is therefore the loci of the points which indicate the condition of steam when condensation has just occurred, and when thermal equilibrium is reestablished at the limit of supersaturation, as *B* in Fig. 7.

Since the supersaturation ratio is defined as the ratio of the actual pressure at a given temperature to the saturation pressure at that temperature, it is necessary to calculate the temperature of the supersaturated steam at the condensation point. This may be done by using the thermodynamic relations for superheated steam. If the subscript 1 is used to denote the initial conditions, and $n = 1.3$ is the index of isentropic expansion for superheated steam,

$$p_1v_1^n = pv^n \quad \text{[14]}$$

$$p_1v_1 = RT_1, \quad pv = RT \quad \text{[14a]}$$

$$\frac{p_1v_1}{T_1} = \frac{pv}{T} \quad \text{[15]}$$

where p, v, and T are the conditions of the supersaturated steam at the condensation point. Combining [14] and [15], we have the well-known isentropic relationship:

$$T = \frac{T_1}{\left(\frac{p_1}{p}\right)^{\frac{n-1}{n}}} \quad \text{[16]}$$

If T_1, p_1, and p are known, the temperature at the condensation point can be calculated from [16], and the pressure p_s corresponding to that temperature can be found from the steam tables. Substituting for n, [16] reduces to:

$$T = \frac{T_1}{\left(\frac{p_1}{p}\right)^{0.231}} \quad \text{[17]}$$

A sample calculation follows:

Initial pressure = 64.7 lb per sq in. abs
Initial temperature = 303.0 F 762.6 abs
Observed condensation pressure = 34.2 lb per sq in. abs

Then

$$\left(\frac{p_1}{p}\right) = 1.901, \text{ and } \left(\frac{p_1}{p}\right)^{0.231} = 1.159$$

$$T = \frac{T_1}{1.159} = \frac{762.6}{1.159} = 658.0 \text{ F abs}$$

and

$$t = (658.0 - 459.6) = 198.4 \text{ F}$$

From Keenan's steam tables, the pressure p_s corresponding to 198.4 F is 11.15 lb per sq in. abs so that the supersaturation ratio

$$S = \frac{34.2}{11.15} = 3.05$$

When the supersaturation ratio has been calculated for a given observation, it becomes possible to apply the von Helmholtz Equation [13], to estimate the radius of the droplets formed when condensation occurs. Thus:

$$\log_e 3.05 = \frac{2\gamma}{rRDT} = 1.1151 \quad \text{[18]}$$

D = density of water at temperature t = 60.1 lb per cu ft at 198.4 F
R = gas constant for superheated steam = 86 (approx.)
T = absolute temperature, F = 658.0
γ = surface tension of water at 198.4 F = 4.13×10^{-3} lb per ft (see Fig. 10).

Then, for $S = 3.05$

$$r = \frac{2 \times 4.13 \times 10^{-3}}{1.1151 \times 658.0 \times 60.1 \times 86}$$

$$= 2.11 \times 10^{-9} \text{ ft} = 6.4 \times 10^{-8} \text{ cm}$$

The value thus obtained for the effective radius of the drops formed in condensation is in close agreement with those found by Wilson and Powell.

6 The Experimental Work

After it had been found that the condensation pressure could be measured with reasonable accuracy, the first feature to be investigated was the variation of that pressure with varying

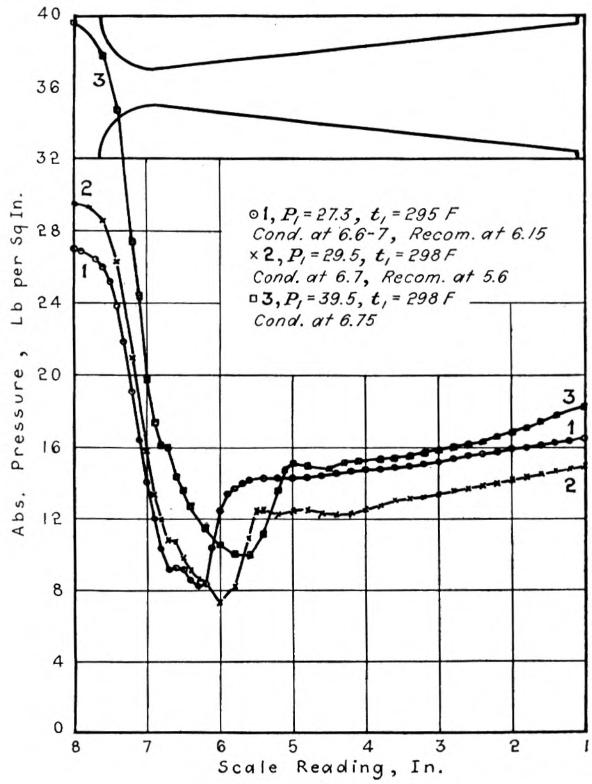

Fig. 8(*a*)

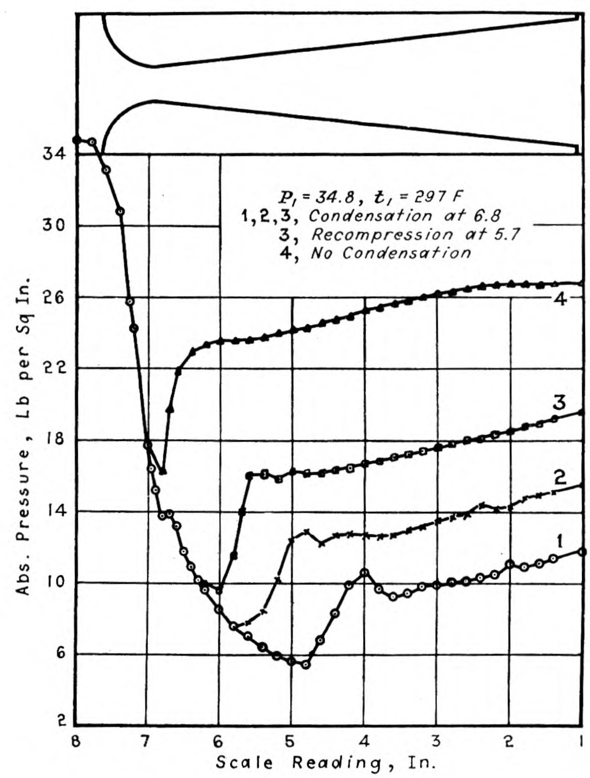

Fig. 8(*b*)

initial conditions. Using a simple convergent-divergent nozzle form, No. 1 in Fig. 5, a series of measurements was made in the manner outlined in Section 4. Inlet pressures varied from 11.0 to 75.0 lb per sq in. abs, the range in which turbine condition curves usually cross the saturation line on the Mollier chart. Above the latter pressure the incoming steam was too wet for satisfactory observation, and it was undesirable to subject the glass plate to unnecessary stress.

Figs. 8(*a*) and 8(*b*) show typical results of the observations with absolute pressures plotted against position along the nozzle axis. In these curves it will be seen that, at the point where condensation occurs, there is an abrupt halt in the fall of pressure. After the condensation point is passed the expansion continues as before. This unexpected feature was at first dismissed as an error in the manometer reading but it was repeated with such consistency that it must be accepted as an actual occurrence. A similar irregularity in the pressure-expansion curve was noticed by Prof. C. A. Robb in his work on recompression in nozzles, carried out at the Johns Hopkins University in 1931–32, but not yet published. The phenomenon was attributed by him to roughness in the nozzle wall, but this is improbable because the irregularity always coincides with the condensation point, and does not remain at the same spot in the nozzle when the pressure conditions are varied. It is probably due to the fact that the rapid increase in the specific volume of the steam, caused by the liberation of the latent heat of the condensed moisture, is not compensated by the increase in velocity and so must result in an increase in pressure, or in sustained pressure with increasing nozzle areas.

The pressure-distance curves, Figs. 8(*a*) and 8(*b*) show that the steam invariably over-expands and then recompresses to the back pressure. When in recompression the steam reached the pressure at which condensation originally occurred, a dark spot appeared in the nozzle, as in Fig. 13(*a*), to be followed once more by the familiar blue of the scattered light if the pressure again fell below the condensation value. The cause for such dark spots was the absence of droplets in the dark region. This disappearance of the droplets when the steam pressure was raised above the condensation value is evidence in favor of the supersaturation theory, which contends that droplets of a given size can exist only when the actual pressure of the surrounding vapor exceeds by a certain amount the saturation pressure of the liquid for the existing temperature. When this excess is not present the droplets should evaporate, and they apparently do so.

The test shown in Fig. 8(*b*) was performed to study the effect of variations in the back pressure, the inlet pressure remaining fixed at 34.8 lb per sq in. abs. In curves Nos. 1, 2, and 3, in which the back pressure was below 20.0 lb per sq in. abs, condensation occurred at the same point in the nozzle and at the same pressure, 13.9 lb per sq in. abs. The abrupt halt in the fall of pressure at the condensation point is evident. In curve No. 3, the recompression reached the condensation pressure and the droplets reevaporated. In curve No. 4, the back pressure was so high that the condensation pressure was not reached and no condensation was seen.

The pressure along a vertical section through the nozzle did not appear to be constant. This was noticeable when different sections of the nozzle were illuminated by the use of a narrow slit through which the light entered the nozzle. The center of the nozzle seemed to be at a higher pressure than the outer portions so that condensation could exist in the outer sections of the steam while the central part might be transparent and hence devoid of condensation. This was a source of error which had to be guarded against because, if the entire cross-section of the steam were illuminated, the condensation might appear to be continuous, whereas in reality, the central portion at which the pressure was being measured might be quite free from condensation. In order to be sure that condensation was actually occurring at the point where the search tube was measuring the pressure, a slit was used to confine the light to the vicinity of the search tube. Fig. 6(*a*) shows the illumination confined to the search tube section, with the search tube and the pressure holes clearly visible. Fig. 6(*b*) shows the effect of illuminating the entire cross-section of the nozzle, with the search tube almost hidden by the mist.

With nozzle No. 1, condensation appeared as a blue mist, and the blue color was maintained throughout the length of the nozzle, which indicates that there was no growth of the droplets during their passage. If growth had occurred, the blue light would have changed in color toward the red, and the intensity of the scattered light would have increased.

TABLE 1 SUMMARY OF RESULTS OF TESTS ON NOZZLE NO. 1

Point	Test no.	p, abs	t_1	Cond. pres.	t_2	S	$r \times 10^8$, cm
A	50–1	64.7	303	34	198.4	3.05	6.69
B	45–2	59.7	297	31	190.4	3.29	6.39
C	40–2	54.8	292	29	188.4	3.22	6.57
D	35–2	49.7	307	22	176.4	3.17	6.86
E	30–2	44.7	303	18.7	164.4	3.56	6.46
F	25–4	39.7	302	16.1	158.4	3.54	6.58
G	20–2	34.7	309	11.5	135.4	4.49	5.90
H	15–2	29.7	286	11.9	144.4	3.69	6.63
I	12.5–1	27.3	295	9.4	128.4	4.31	6.19
J	11–1	25.9	295	8.75	126.4	4.34	6.24
K	5–3	20.0	289	6.3	113.4	4.54	6.25
L	2.7–1	17.5	295	5.3	112.6	3.89	6.97
M	0–1	14.4	270	4.4	94.4	5.51	5.87
N	0–3	12.8	285	3.3	84.4	5.64	5.95
O	0–2	11.3	283	2.9	82.4	5.30	6.17
S	Stodola	71	Sat.	38.2	303	3.1	....

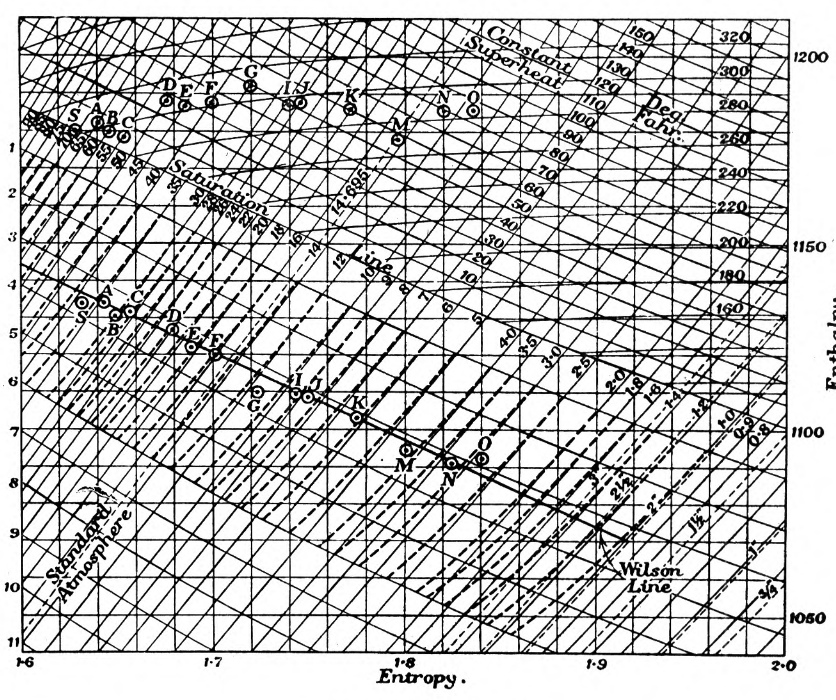

FIG. 9 WILSON LINE
(From tests made on nozzle No. 1.)

The data from the tests on nozzle No. 1 are tabulated in Table 1, which also contains the results of the calculations of the supersaturation ratio and the droplet radius for each condition. On Fig. 9, a revised Keenan Mollier chart, the condensation points found with nozzle No. 1 are located in the manner described in Section 4. The condensation points lie between the 3 and 4 per

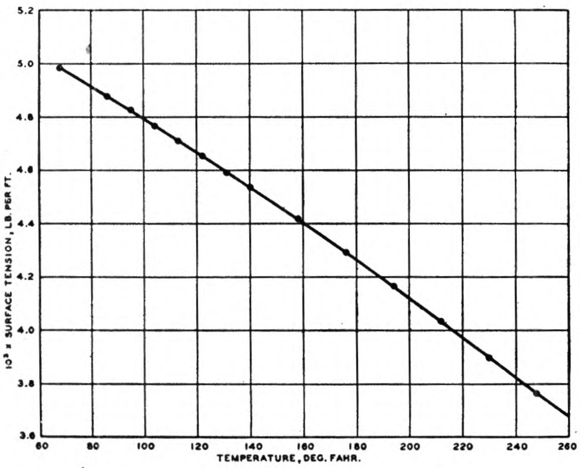

FIG. 10 VARIATION OF SURFACE TENSION OF WATER WITH TEMPERATURE
(Data from International Critical Tables.)

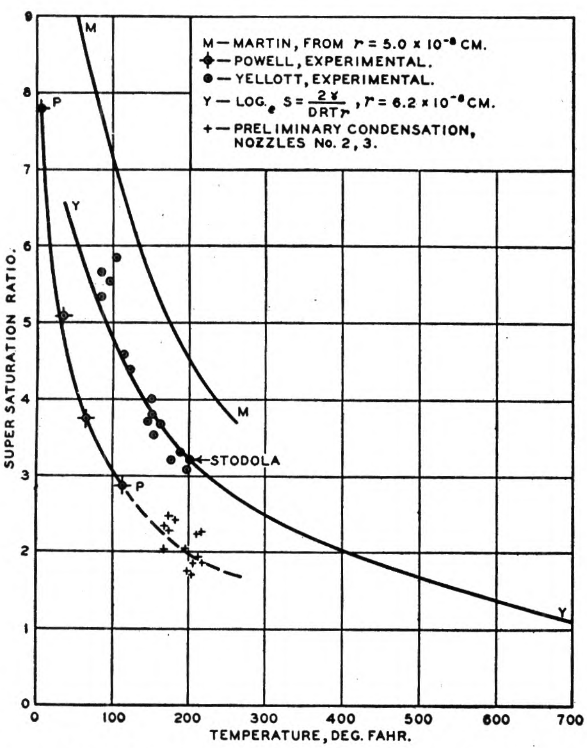

FIG. 11 DETERMINATION OF SUPERSATURATION RATIO AT THE WILSON LINE

cent moisture lines, and the line which is faired through them will be referred to as the Wilson line. Point S at the high pressure end of the Wilson line is the point found by Stodola.

It may be objected that there is a possibility of condensation occurring in droplets too small to be seen. This is improbable, because particles of any finite size, including molecules, scatter light to a noticeable extent. The scattering of light by steam molecules was studied to establish this point and it was found by another experiment that a beam of concentrated arc light could be plainly seen in an atmosphere of superheated steam, just as a searchlight beam can be seen in the sky at night. The scattered light was dark blue in color, and, although faint, was bright enough to be visible against a black background. Whenever a drop of water crossed the beam, the light scattered by it stood out with great intensity. From this experiment, as well as from the work of many physicists (50) it may be concluded that any droplets resulting from condensation will be visible.

In Fig. 11 the supersaturation ratios at various temperatures are plotted against those temperatures. A mean value of the drop radius in Table 1, calculated by the von Helmholtz equation, is about 6.2×10^{-8} cm, which agrees with the values of Wilson, Powell, and Stodola. If this value is substituted in Equation [13], we have:

$$\log_e S = \frac{2\gamma}{D \times 86 \times T \times \dfrac{6.2 \times 10^{-8}}{2.54 \times 12}} = 1.15 \frac{\gamma}{D \times T} \times 10^7$$

Taking the proper values of γ, the surface tension, from Fig. 10, and of D, the water density, from Keenan's steam tables, and solving for S at various temperatures, curve (Y) in Fig. 11 is obtained. This curve is a fair representation of the experimental points, since as many lie above the curve as below it, and a reasonable number lie directly on it. Curve (P) presents data obtained by Powell in his experiments, and curve (M) values derived by Martin from his substitution of Callendar's value of r in the von Helmholtz equation. The results of the present

Fig. 12(a)

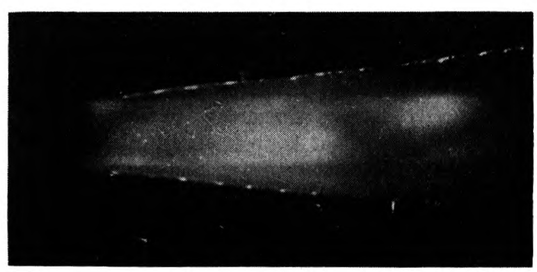

Fig. 12(b)

FIG. 12 FLOW THROUGH NOZZLE NO. 1
[a (upper)—Photographed through a Nicol prism, showing photoelastic effect. Initial conditions: 41 lb per sq in. gage, 290 F, 18 lb per sq in. abs back pressure. b (lower)—Prism turned slightly to show strain lines near throat. Conditions same as in case (a).]

investigation lie between those of Martin and Powell. The Wilson line, plotted in Fig. 9, also lies between their versions of the line and is closer to that of Martin.

A drop radius of 6.2×10^{-8} cm, while only approximate, is of a reasonable order of magnitude. The nature of the light scattered by the droplets is evidence in favor of a very small droplet size. (The optics of small particles is presented briefly in Appendix 1.) The intensity of the scattered light is symmetrical, resembling that shown in (*a*), Fig. 20. The light scattered at 90 deg to the incident light is completely plane polarized. The color of the scattered light is sky-blue. These three facts definitely associate the droplets with Rayleigh's theoretical infinitely small dielectric spheres.

Scattered light of this character can come only from particles whose radius is many times smaller than the wave-length of the light which falls upon them. It can only be estimated how much smaller they are than that wave-length, but the character of the scattered light suggests that the wave-length is from 100 to 1000 times as great as the radius of the drops. Assuming 6.0×10^{-5} cm as an average value of the wave-length of arc light, the optical evidence indicates that the radius is in the neighborhood of 6.0×10^{-8} cm, which is in good agreement with the value of

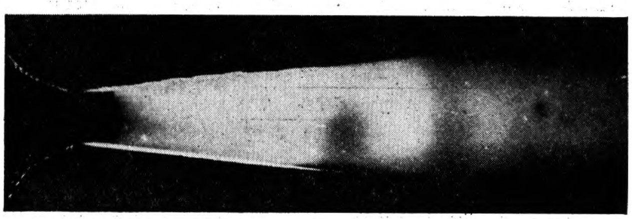

Fig. 13(*a*)

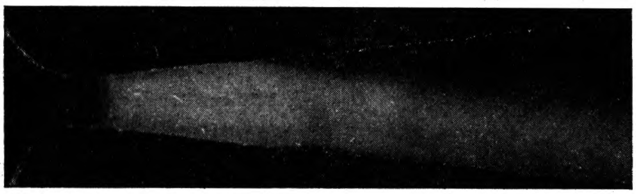

Fig. 13(*b*)

FIG. 13 FLOW THROUGH NOZZLE NO. 1

[*a* (upper)—Completely illuminated; dark bands indicate recompression above condensation pressure with evaporation of droplets. Initial conditions: 20 lb per sq in. gage, 287 F, 15 lb per sq in. abs back pressure. *b* (lower)—Photographed after pressure raised to 35 lb per sq in. gage, 23 lb per sq in. abs back pressure. Steam flow breaks away from one side of nozzle and continues down the other.]

the radius as calculated from the von Helmholtz equation. A reasonable figure for the radius of the droplets formed by the condensation of steam in a rapid expansion is therefore 6.2×10^{-8} cm or 2.03×10^{-9} ft.

The polarization of the scattered light gives rise to another interesting phenomenon. Since the glass plate which forms the top of the nozzle is under strain, stresses result, and the glass becomes doubly refractive, assuming photoelastic properties. Thus if the scattered light is observed through a Nicol prism, brilliant colors are visible, corresponding to the stresses in the glass. Figs. 12(*a*) and 12(*b*) were taken through a large Nicol prism, and, although the colors cannot be distinguished, dark lines can be seen which represent strains in the glass. These strain lines are probably caused by standing waves in the steam.

Several interesting features of steam flow were discovered with nozzle No. 1. Recompression and the breaking away of the jet from the nozzle walls, predicted by Stodola (29, p. 93), were seen and photographed. In Fig. 13(*a*) the steam breaks away from both sides of the nozzle during recompression, and then reexpands to fill the nozzle, while a second recompression is evidenced by a second dark band. In Fig. 13(*b*) the steam breaks away from the sides, but instead of returning to both walls, it continues down one side. This condition was very unstable, the jet alternating rapidly from one side of the nozzle to the other.

The observation of liquid water in steam was fully discussed by Thomas (31) and his work is substantiated by this investigation. It is possible to see drops of water in flowing steam and the absence of moisture can be detected by the transparency of the steam. The light scattered by the steam molecules is so faint that it can be seen only against a perfectly black background; hence dry steam appears transparent. The moisture entrained in the entering steam appears as relatively large drops, a few hundredths of an inch in diameter. As the steam is accelerated through the nozzle, however, the large drops are broken up into much smaller droplets in the manner explained by Soderberg (28). This feature was not fully investigated in the present research and Soderberg's paper suggests an interesting problem which could be attacked profitably with the methods used in this work.

Summarizing the results so far presented, it is very improbable that any condensation will occur in the expansion of steam through a convergent-divergent nozzle of the type used in these tests until the steam has reached the condition approximately represented by the region between the 3 and 4 per cent moisture lines on the Mollier chart. When this region is reached, condensation apparently takes place on a vast number of tiny nuclei. The radius of the drops thus formed appears to be about 6.2×10^{-8} cm. When the steam in recompression again reaches

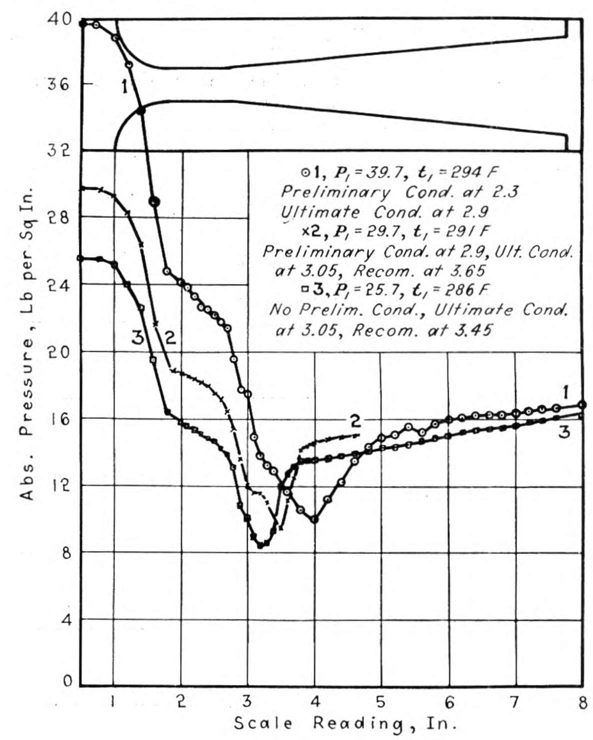

FIG. 14

the pressure at which condensation occurred, most, if not all, of the droplets seem to reevaporate, appearing again if the pressure once more falls below the condensation value. Only blue light, completely plane polarized when observed at right angles to the incident light, was to be seen in the nozzle. It is highly probable therefore that there is no growth of the droplets as they pass through the nozzle.

The effect of supersaturation upon the flow of steam has been treated in detail by Stodola (29), Goodenough (16), and others. It may be concluded that supersaturation always occurs in the expansion of saturated steam, and for that reason the formula for superheated steam should be used to calculate the flow of saturated or slightly wet steam through nozzles. The actual effect of supersaturation in turbine operation is small, but, as has been pointed out previously, there is a certain loss of availability caused by the increase of entropy which accompanies condensation and the establishment of thermal equilibrium. This loss may amount to as much as 4 per cent of the isentropic enthalpy drop from the saturation line to the Wilson line.

Prof. J. H. Keenan of the Stevens Institute of Technology suggested that nozzle No. 1 be replaced by another designed to give a less rapid expansion in the region where supersaturation occurs. The nozzle chosen for this purpose, No. 2 in Fig. 5, had a rounded inlet identical to that of No. 1, but a 1.0 in. straight section was interposed between the convergent and the divergent portions. As shown in Fig. 14, this design produced the desired effect, for there was a rapid expansion through the rounded section, followed by a relatively slow drop in pressure through the straight throat. Additional expansion and recompression took place in the diverging portion.

The condensation which occurred in nozzle No. 2 differed radically in appearance from that encountered with No. 1. Observations of the flow through nozzle No. 1 showed that there was a distinct and relatively sharp curved line, concave toward the high-pressure end, which marked the beginning of condensation. A line of demarcation was thus provided between the transparent superheated steam and the misty wet steam. With nozzle No. 2 the condensation curve again appeared and the pressures measured at that curve for varying initial conditions were found to lie on the Wilson line shown on Fig. 9. The term "ultimate condensation" will be applied to that which occurs at the Wilson line to distinguish it from the "preliminary condensation" which is about to be discussed.

The concave curve in nozzle No. 2 was preceded by a slight condensation which began as a trace of bluish haze and rapidly became more dense as the ultimate-condensation curve was approached. Fig. 15 shows a photograph of the flow in nozzle No. 2 obtained by focussing the light from the arc at the low-pressure end into a sharp beam. The preliminary condensation can be seen and the curved line which denotes ultimate condensation is also visible, although slightly different in shape from that in Fig. 6.

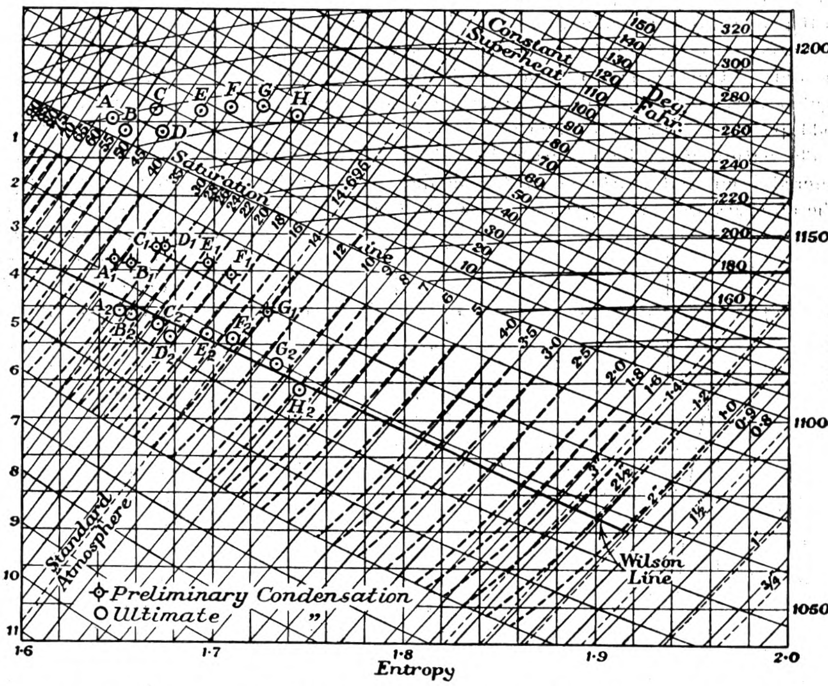

FIG. 16 PRELIMINARY AND ULTIMATE CONDENSATION FROM TESTS ON NOZZLE NO. 2

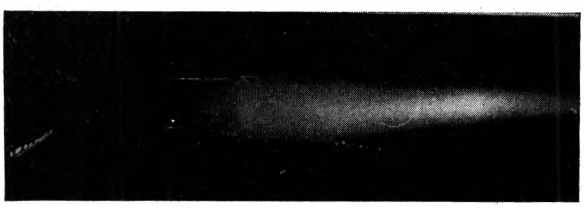

FIG. 15 FLOW THROUGH NOZZLE NO. 2

(Illuminated from low-pressure end by sharply focussed beam. Preliminary condensation can be seen before curved line denoting ultimate condensation. Initial conditions: 28 lb per sq in. gage, 288 F, 15 lb per sq in. abs back pressure.)

TABLE 2 SUMMARY OF RESULTS OF TESTS ON NOZZLE NO. 2

Point	Test no.	p, abs	t	Cond. pres.	t_2	S	$r \times 10^8$, cm
			Preliminary Condensation				
A_1	45–1	59.8	298	37.0	218.0	2.24	8.75
B_1	40–1	54.8	292	34.0	214.0	2.23	8.86
C_1	35–1	49.8	297	31.0	219.0	1.86	11.28
D_1	30–1	44.8	285	29.0	213.0	1.94	10.67
E_1	25–1	39.8	295	23.0	204.0	1.84	11.98
F_1	20–1	34.6	293	19.5	199.0	1.74	13.28
G_1	15–1	29.8	291	14.0	170.4	2.32	9.52
			Ultimate Condensation				
A_2	45–1	59.8	298	30.0	183.4	3.54	6.17
B_2	40–1	54.8	292	27.9	175.0	4.11	5.93
C_2	35–1	49.8	297	23.5	183.4	2.94	7.15
D_2	30–1	44.8	285	21.0	164.4	3.99	5.89
E_2	25–1	39.8	295	17.5	164.4	3.33	6.78
F_2	20–1	34.8	293	15.0	159.4	3.21	7.11
G_2	15–1	29.8	291	9.0	98.4	9.88	...

In order to study this preliminary condensation, of which no trace had been found in nozzle No. 1, a series of tests was made as before with inlet pressures ranging from 10.0 to 45.0 lb per sq in. gage. Fig. 14 shows typical pressure-distance curves, while Table 2 includes the data and the results of the calculations of supersaturation ratios and droplet radii.

Fig. 16 shows the data plotted on the revised Mollier chart from which it is evident that while ultimate condensation occurs approximately at the Wilson line, preliminary condensation appears to take place at points scattered about the 2 per cent moisture line. Because of the nebulous character of the earliest traces of the preliminary condensation, it is difficult to measure the exact pressure at which it begins. This difficulty accounts for the wide variations in the droplet radii recorded in Table 2 as well as for the scattering of the points in Fig. 16.

As the droplets formed by preliminary condensation approached the line of ultimate condensation, the light scattered by the droplets changed rapidly in color from the initial blue through green and yellow to red. The red light was immediately lost in the more intense blue light scattered by the myriad of droplets formed at the curve of ultimate condensation. When the blue color disappeared because of recompression of the steam above the ultimate condensation pressure, the red again became visible. Analysis of these facts indicates that the preliminary droplets probably begin as very minute particles, about 10.0×10^{-8} cm in radius, but grow with extreme rapidity until they approach the size of red light waves, about 6.0×10^{-5} cm. Further growth is apparently halted by the crossing of the ultimate condensation curve, for the nuclei which come into action in vast numbers at that line seem to acquire all of the available moisture. Coalescence of these drops is probably prevented by the high velocity at which they are traveling. Soderberg (28) presents curves which indicate that with steam velocities above 1200 fps the maximum drop size cannot exceed 2.5×10^{-5} cm if the specific volume of the steam is below 50.0 cu ft per lb. While this limit is certainly of the correct order of magnitude, it is perhaps too low because the drops which caused the red light in nozzle No. 2 were undoubtedly as large as red light waves, although the steam velocity was approximately 1600 fps and the specific volume about 20.0 cu ft per lb.

The nature of the light scattered by the preliminary droplets differed from that scattered by the smaller ultimate droplets. The intensity was much greater in the direction from which the light came and the polarization was not complete. Moreover, the colors other than blue were not visible unless the angle of observation was less than 90 deg to the incident light. It is therefore probable that the colors were produced by a complex interaction of scattering, refraction, and interference, and it is impossible to place an accurate limit on the size of the drops. The fact that the red light was partially polarized, however, indicates that 6.0×10^{-5} cm, or 2.3×10^{-5} in., is a reasonable value for the radius of the largest drops observed in nozzle No. 2.

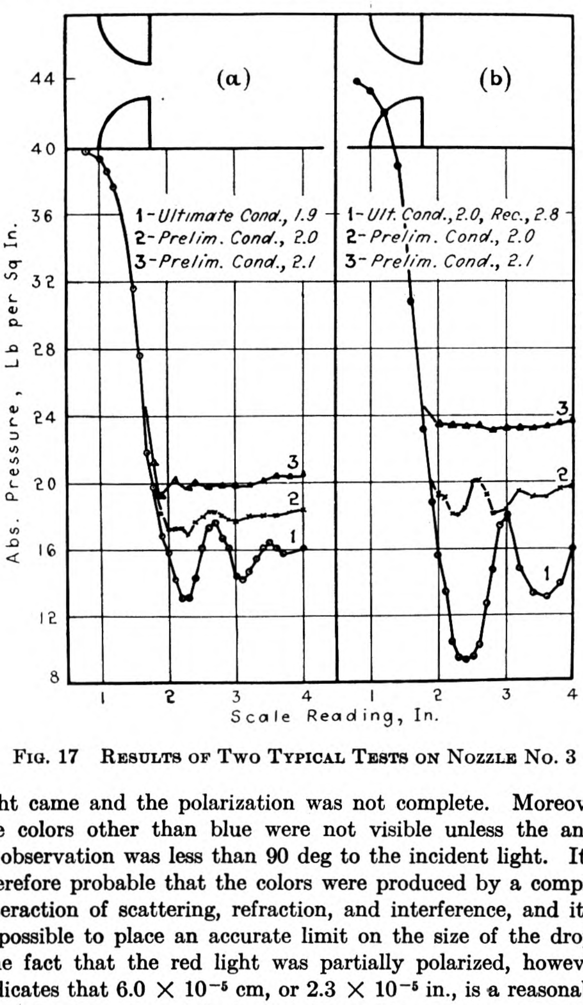

FIG. 17 RESULTS OF TWO TYPICAL TESTS ON NOZZLE NO. 3

The existence of preliminary condensation raised some interesting questions. Stodola found no traces of such condensation nor did the theoretical investigators consider the possibility of its existence. Up to the present time it has been assumed that supersaturation, if it existed at all, would continue until the Wilson line was reached, at which point condensation would occur and equilibrium would be reestablished. Dust or other foreign particles in the steam would cause the type of condensation mentioned above, but particles large enough to act as nuclei would also scatter light with visible intensity before any condensation occurs. No such particles were seen in any of the tests.

It was hoped that preliminary condensation could be proved to occur upon ions, in which case the conventional theory would be vindicated. Condensation earlier than the ultimate was found by Wilson (34), Thomson (33), and Barus (7, 8) when electrified nuclei were present. In order to study the possible electrification of the droplets, nozzle No. 2 was replaced by No. 2B (Fig. 5), made of bakelite with brass plates set into the straight sections to act as electrodes. When the steam was flowing and condensation was occurring as in Fig. 15, a potential of 110 volts, dc, was connected across the electrodes. There was no apparent change in the character of the condensation, nor did reversing the polarity of the electrodes have any visible effect. If the drops were electrified, it would be supposed that the positive drops would be attracted to the negative electrode, and vice versa. There was, however, no evidence of attraction or repulsion. This test was by no means conclusive because the voltage was not high enough to produce a very vigorous attraction and the high velocity of the water particles might have obscured any effect which existed.

It was decided to apply another test. If preliminary condensation occurred on ions, increasing the number of ions should increase the density of the mist which indicated condensation. A similar experiment had been performed successfully by Wilson and Thomson. The most prolific source of ions which could be obtained was the spark from an induction coil. An insulated wire was led into the nozzle and so arranged that the spark could jump from the wire to the electrodes or to the bottom of the nozzle. When the coil was in operation a strong spark resulted but there was no visible effect upon the condensation. This is directly opposed to the evidence of von Helmholtz (19) who found that electrification of a steam jet increased the condensation. The duration of the spark from an induction coil, about 10^{-6} sec, is probably so short that any effect which occurred might not be visible because of the high velocity of the steam. The experiments to discover whether the droplets are electrified must therefore be regarded as inconclusive and it is hoped that more work can be done on this phase of the subject.

To develop a theory which will explain both the preliminary and the ultimate condensation, the tests on nozzle No. 2 must be analyzed. A very significant feature is that with an initial pressure of 10.0 lb per sq in. gage and a back pressure of 1.0 lb per sq in. gage, represented by point H in Fig. 16, no preliminary condensation was seen. When the inlet pressure was raised to 15.0 lb per sq in. gage, point G, preliminary condensation was visible. The only apparent difference between these two conditions was that the velocity of the steam at the condensation point was higher in the first case, about 1700 fps, when no pre-

TABLE 3 SUMMARY OF TESTS ON NOZZLE NO. 3

Point	Test no.	p, abs	t	Cond. pres.	t_2	S	$r \times 10^6$, cm
			Preliminary Condensation				
$A_{1,2}$	5–1,2	20.1	284	9.55	166	1.75	14.5
$B_{1,2}$	10–1,2	24.5	289	11.65	168	2.84	11.3
C_1	13–1	29.5	294	12.5	158	2.77	8.17
C_2	13–3	29.5	290	14.4	173	2.27	9.77
D_1	20–1	34.6	294	19.6	201	1.70	13.8
D_2	20–2	34.6	297	17.0	176	2.47	8.76
E_1	25–2	39.7	298	19.7	184	3.01	7.93
F_1	30–3	34.7	303	19.5	184	2.40	8.79
G_1	35–3	49.7	307	25.5	198	2.04	10.5
H_1	40–3	54.7	307	30.7	218	1.86	11.3
			Intermediate Condensation				
E_2	25–2	39.7	298	18.0	170	3.01	7.89
F_2	30–2	44.7	303	19.5	172	3.10	7.04
G_2	35–2	49.7	303	22.0	172	3.49	6.88
H_2	40–2	54.7	305	28.0	195	2.69	7.57
			Ultimate Condensation				
A_3	5–3	20.1	287	6.0	104	5.60	5.59
B_3	10–3	24.5	290	7.5	114	5.26	5.65
C_3	13–3	28.9	297	9.5	139	3.38	7.03
D_3	20–3	34.9	299	13.0	144	4.07	6.17
E_3	25–1	39.7	303	16.4	162	3.30	6.88
F_3	30–1	44.7	306	18.0	169	3.08	7.16
G_3	35–1	49.7	300	22.5	172	3.60	6.22
H_3	40–1	54.7	305	25.0	171	3.92	5.86

liminary condensation occurred, than in the second, about 1500 fps.

The factor controlling the type of condensation to be expected under various conditions may be a function of the rate of change of pressure with time. Tests carried out on nozzle No. 3 (Fig. 5), a simple rounded orifice, lend support to this theory. A series of tests was made with this nozzle using inlet pressures up to 40.0 lb per sq in. gage and the back pressure was varied in each test. Typical results are presented in Fig. 17 and the data are tabulated in Table 3.

It was found that the nature of the condensation in the jet varied with changes in the back pressure. If the steam expanded to a pressure below that of ultimate condensation, for the given initial conditions, the jet appeared as in Fig. 18(*a*). There was no preliminary condensation and the concave ultimate condensation curve of nozzle No. 1 was again visible. In such expansions there was no color other than the blue and the general nature of the condensation was similar to that found in nozzle No. 1. The jet expanded freely, bending first toward one side of the channel and then toward the other. The wavy outlines of the jet corresponded to the oscillations in the pressure-length curves of Fig. 17. When the inlet pressure was raised to 40.0 lb per sq in. gage, the steam expanded to fill the entire channel, Fig. 18(*b*) and the dark bands due to recompression above the ultimate condensation pressure were quite distinct. Fig. 19 shows the data obtained from these tests plotted on the revised Mollier chart.

Fig. 18(*a*)

Fig. 18(*b*)

FIG. 18 FLOW THROUGH NOZZLE NO. 3

[*a* (upper)—Initial Conditions: 25 lb per sq in. gage, 300 F, 16 lb per sq. in. abs back pressure. *b* (lower)—Photographed after pressure raised to 40 lb per sq in. gage. Jet expands to fill entire channel. Dark bands caused by recompression above ultimate condensation pressure.]

When the back pressure was raised until the ultimate condensation pressure was not quite reached during the expansion, the appearance of the jet changed completely. The well-defined wavy outlines disappeared and the initial blue changed to a whitish blue, indicative of larger droplets. This condensation was apparently an intermediate type, being neither preliminary nor ultimate, but resembling each in some particulars. The pressures at which this intermediate condensation took place were slightly above the pressures at the Wilson line.

When the back pressure was raised to a still higher value, preliminary condensation occurred. The condensation first appeared as a bluish mist but the scattered light changed rapidly,

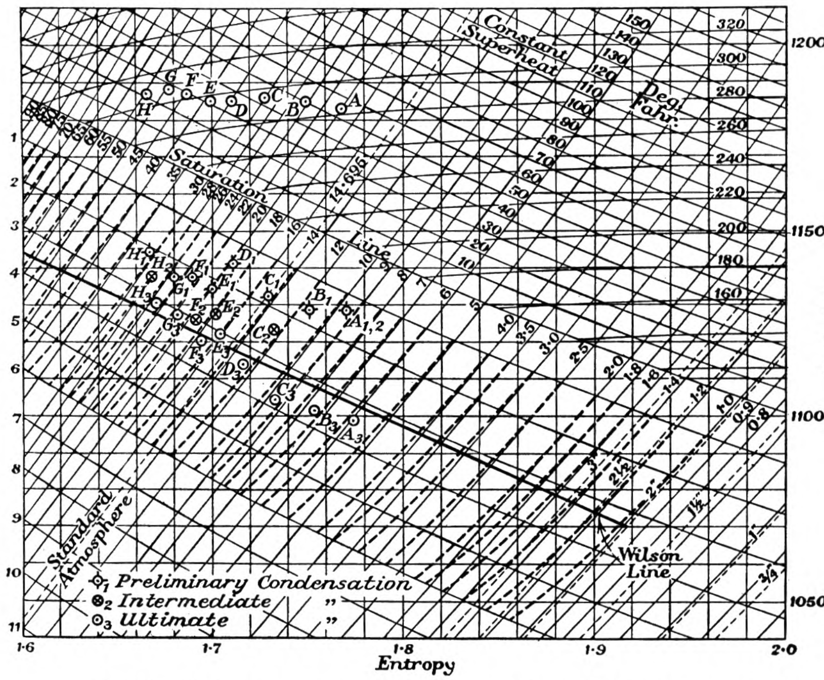

FIG. 19 PRELIMINARY, INTERMEDIATE AND ULTIMATE CONDENSATION FROM TESTS ON NOZZLE NO. 3

through green and yellow to red. The red light was still partially polarized, indicating that the droplets were of the same order of magnitude as the wave length of red light.

The supersaturation ratio calculated for the conditions of preliminary condensation varies from 2.24 for the highest inlet pressures to 1.74 for the lowest. The droplet radius at the beginning of preliminary condensation ranges from 9.52 to 13.3 $\times$ 10^{-8} cm. The wide variations of droplet radius shown in Table 3 are probably due to the difficulty in determining the exact pressure at which preliminary condensation occurred. When the values of the supersaturation ratio at preliminary condensation are plotted against temperature as in Fig. 11, it is found that they lie well below the curve for the ratio at ultimate condensation, but that Powell's curve, if extended, will pass through the group. It is also evident from Figs. 16 and 19 that the preliminary condensation points are located near the 2 per cent moisture line, along which fell Powell's version of the Wilson line (27A). It seems quite probable that the condensation observed by Powell was a form of preliminary, rather than of ultimate, condensation.

The type of condensation which occurred in nozzle No. 3 for any given inlet conditions could be varied among the three types mentioned above by varying the back pressure. The nature of the condensation was apparently a function of the velocity of the steam and of the duration of the condensation process. If the expansion was rapid and continuous to the ultimate condensation condition, no preliminary condensation occurred, and at the Wilson line a vast number of tiny droplets was formed which did not appear to increase in size with further expansion. If, on the other hand, the expansion was slow or the back pressure too high, some preliminary condensation occurred at a pressure higher than that at the ultimate condensation condition. The drops thus formed were larger than those formed at the Wilson line, and they continued to grow as they passed through the nozzle. Growth was apparently halted when the drops reached the order of magnitude of red light waves.

7 Results and Conclusions

The primary object of this investigation was to check the location of the limit of supersaturation, which, on the Mollier chart, is known as the Wilson Line. The secondary object was to determine the size of the water droplets formed by condensation from the supersaturated state, as their size may have some bearing on the erosion of the low-pressure blades in steam turbines.

The Wilson line was located by measuring the pressures at which condensation occurred in an illuminated nozzle. A series of such measurements, covering the range of 10.0 to 75.0 lb per sq in. abs in which turbine condition curves cross the saturation line, resulted in a number of points, through which the Wilson line was drawn. This will provide working data for turbine designers.

The Wilson line lies between the 3 and 4 per cent moisture lines on the Keenan Mollier chart, slightly higher than the original Wilson line of H. M. Martin. Callendar concluded from data on the Lusitania turbines that the Wilson line approximately coincided with the 3 per cent moisture line, and this investigation indicates that he was nearly correct in this assumption.

The radius of the droplets which are formed when condensation occurs at the Wilson line is approximately 6.2×10^{-8} cm, a value determined by the von Helmholtz equation and substantiated by the blue color and complete plane polarization of the light scattered by the droplets.

The theory of supersaturation is verified for rapid expansions through simple convergent-divergent nozzle forms, for not only do the droplets appear at the pressures theoretically predicted, but also they disappear under recompression conditions in a manner which can only be explained by the supersaturation theory.

It was found that the behavior of steam in the illuminated nozzle could be seen clearly and the phenomena of shock, recompression, and the breaking away of the jet from the nozzle walls were observed and photographed.

A second series of experiments on modified nozzles revealed that under certain circumstances condensation could occur before the Wilson line is reached in an expansion. The upper limit of this preliminary condensation region is close to the 2 per cent moisture line on the Mollier chart. Due to the difficulty in distinguishing the first traces of preliminary condensation, the experimental points obtained in the study of this type are too widely scattered to permit definite conclusions to be drawn. Evidence indicated that the velocity of the steam in the condensation region is the controlling factor. The tests for electrification of the droplets, although inconclusive, indicate that electrical methods for removing moisture from the low pressure sections of steam turbines are not likely to succeed.

With nozzle No. 1, the evidence is reasonably conclusive that there is no growth of the droplets during their passage through the nozzle. Growth of the droplets would cause the nature of the scattered light to undergo drastic changes which could easily be observed. The color, intensity, and polarization of the scattered light remain constant throughout the length of the nozzle, however, and there could have been no appreciable change in the size of the droplets.

With nozzles Nos. 2 and 3 it was found that the drops which are formed in preliminary condensation grow very rapidly from their original radius of about 10.0×10^{-8} cm to a magnitude comparable to the wave-length of red light, about 6.0×10^{-5} cm in radius. If, however, the preliminary condensation is followed by ultimate condensation, the first droplets cease to grow when the latter occurs, probably because all of the available moisture is acquired by the vast number of nuclei which become effective at the Wilson line. It may be concluded that no additional nuclei beyond those present in the dry steam are needed to effect complete condensation.

As yet no explanation can be given for the existence of this preliminary condensation. It was found to occur only when the expansion in the supersaturated region was slow or interrupted. This suggests that long parallel sections of converging nozzles in the low pressure stages of steam turbines may encourage preliminary condensation and the subsequent formation of droplets large enough to cause erosion of the blades upon which they are discharged.

Appendix 1 contains a brief presentation of the optics of small particles, and the construction of the revised Keenan Mollier chart is described in Appendix 2. A bibliography of the leading articles on supersaturation and related subjects will be found in Appendix 3.

Acknowledgments

This research was carried on by the author at the Johns Hopkins University during two years of graduate study in mechanical engineering, under the direction of Prof. A. G. Christie, who rendered invaluable assistance in the experimental work and in the preparation of this paper. The author wishes to acknowledge his indebtedness for the helpful contributions of President J. S. Ames of that University, of Drs. A. H. Pfund, R. W. Wood, and R. B. Barnes of the Physics Department, and of Associate Professor J. C. Smallwood of the Mechanical Engineering Department. Prof. J. H. Keenan of the Stevens Institute of Technology offered many valuable suggestions, and his interest is greatly appreciated.

Appendix 1

THE OPTICS OF SMALL PARTICLES

For the purpose of optical study, small particles must be divided into two classes, those smaller than and those larger than the wave-length of light. Particles whose diameter is less than the wave-length of the light which falls upon them give rise to the phenomenon of scattering, which will be considered in detail since the droplets encountered in this work seem to be of that order of magnitude. Larger particles can be studied by means of the light reflected or refracted by them.

The scattering of light by small particles was studied by Lord Rayleigh (34, 35, 36), who developed the following formula for the intensity of the light scattered by dielectric spheres, the radius of which is many times smaller than the wave-length of the light which falls upon them. If I is the intensity of the scattered light,

$$I = k \frac{r^6}{\lambda^4} (1 + \cos^2 \beta) \qquad [18]$$

where k = a constant
r = radius of the spheres
λ = wave-length of the incident light
β = angle between the line of observation and the incident light

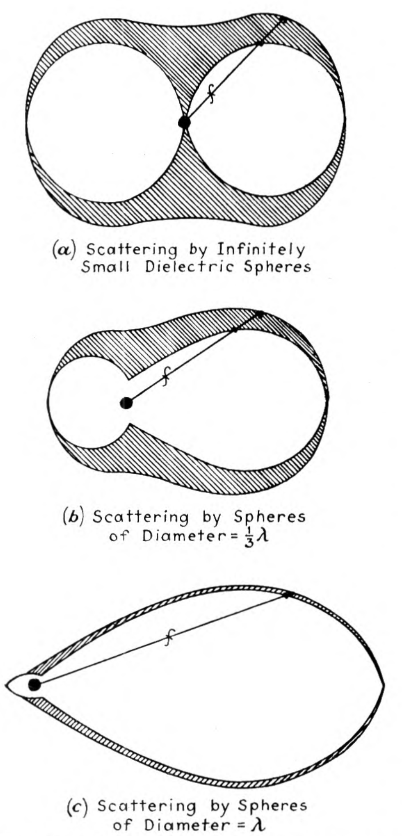

FIG. 20 GRAPHICAL REPRESENTATION OF LIGHT SCATTERED BY SPHERES
(Sizes ranging from infinitely small to a diameter equal to the wave-length of the incident light.)

This formula gives a symmetrical distribution of the scattered light, with the intensity twice as great in the direction from which the light comes as in a direction normal to it.

The light which is scattered at right angles to the incident light is completely plane polarized, and can be extinguished if the incident light is properly polarized. This polarization is the distinguishing feature of scattered light.

It will be seen from Equation [18] that, at a given angle of observation, the intensity of the scattered light varies with the sixth power of the radius of the sphere. Thus a small increase in the radius will cause a large increase in the intensity. Likewise, the intensity of the scattered light varies inversely as the fourth power of the wave-length, so for a given particle size the short wave-lengths will be scattered much more vigorously than the long and the scattered light will be blue in color. This blue color is the characteristic by which light scattered by very small particles can be identified.

There is no lower limit in size below which particles will not scatter light. Any particle of finite radius will scatter light to some degree, and the scattering can usually be observed if the proper care is taken. Strutt (58) has studied the scattering of light by the molecules of many gases. He found that the scattered light when examined spectroscopically was the same as the incident light except that the longer wave-lengths had been removed.

Mie (50), Shoulejkin (57), and Blumer (41–44) have studied the scattering of light by particles of all sizes. The work of Shoulejkin is of particular interest here because he has calculated the intensity of the light scattered by small dielectric spheres, in which class water droplets must be considered. He has represented graphically in polar coordinates the intensity of the light scattered by spheres ranging in size from infinitely small to equal to the wave-length of the incident light. In (*a*), (*b*), and (*c*), Fig. 20, the total length of any vector, as *f*, represents the total intensity of the light scattered along that vector, the incident light coming from the left. The portion of the vector between the inner and the outer curves represents the fraction of the light which is polarized. For infinitely small spheres, the scattering, following the Rayleigh formula, is symmetrical, and the polarization is complete at an angle of 90 deg to the incident light, as in (*a*).

In the radiation from particles whose diameter is about 1/3 the wave-length of light, as in (*b*), the polarization is nowhere complete and its maximum is shifted from 90 deg to the higher angles. The intensity is no longer symmetrical but is noticeably greater in the direction from which the light comes.

With particles of diameter equal to the wave-length, as in (*c*), the assymmetry is still greater, and the intensity is far greater in the direction of the incident light than in the opposite direction. The scattered light is only slightly polarized.

According to Shoulejkin, the nature of the scattered light does not change until the dimensions of the particles have become so great that one can just begin to study their behavior by applying the laws of interference, refraction, and reflection of light. Such particles are of microscopic dimensions and can be measured by various methods, an excellent discussion of which is given by Whytlaw-Gray and Patterson (60).

Appendix 2

A METHOD OF COMPUTING THE PROPERTIES OF SUPERSATURATED STEAM

It has been found that, in the region near saturation, the isentropic expansion of superheated steam can be expressed with a high degree of accuracy by the equation:

$$p_1v_1^{1.3} = p_2v_2^{1.3} \qquad [19]$$

where subscripts 1 and 2 denote the initial and final conditions, respectively.

Since supersaturated steam expands in the same manner as superheated, it may be assumed that the same law is applicable to both. If, therefore, a given superheated condition is assumed,

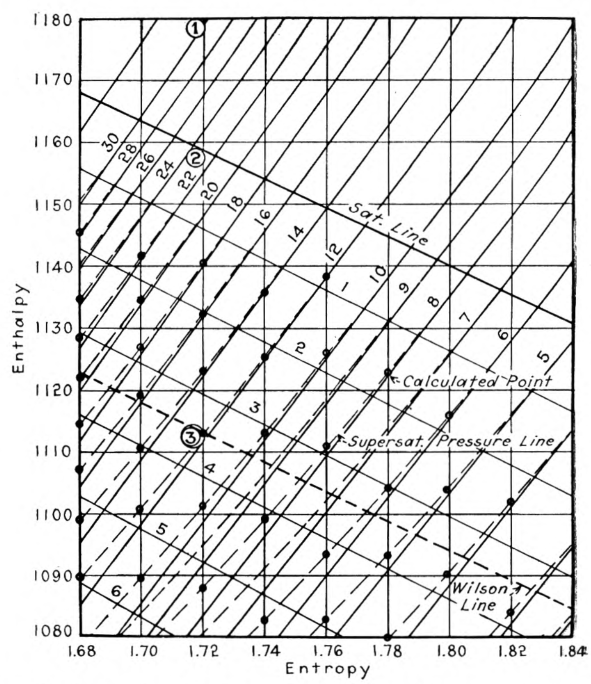

FIG. 21 LARGE-SCALE REPRODUCTION OF PORTION OF REVISED KEENAN CHART

it is possible to calculate the specific volume of superheated or supersaturated steam at the same entropy and at any desired lower pressure, from the equation:

$$v_2 = v_1\left(\frac{p_1}{p_2}\right)^{0.77} \qquad [20]$$

Since the work done in an adiabatic isentropic expansion is equal to the change in enthalpy,

$$h_2 = h_1 - \left(\frac{144}{778}\right)\left(\frac{1.3}{1.3-1}\right)(p_1v_1 - p_2v_2)\ldots\ldots [21]$$

where p_1 and p_2 are in lb per sq in., and v_1 and v_2 are in cu ft per lb. Since h_1, p_1, v_1, and p_2 are known, and v_2 can be calculated from Equation [20], it is possible to compute h_2 for any desired lower pressure.

Over a small range we may apply the perfect gas relationship

$$\frac{p_1v_2}{T_1} = \frac{p_2v_2}{T_2}$$

to calculate T_2 and consequently t_2. Thus all of the properties of supersaturated steam can be obtained for any desired pressure.

In order to plot the revised Keenan chart used in Fig. 9, it was necessary to have at least five entropy-enthalpy values for each pressure line. Accordingly, eighteen points were chosen for entropies between 1.60 and 1.94. By calculating the isentropic enthalpy drop from each initial point to successively lower pressures, the required points were obtained. Fig. 21, a large scale reproduction of a small portion of the revised chart, shows the method of drawing the supersaturated pressure lines.

The accuracy with which this method can be applied is illustrated by the following sample calculation, in which the calculated values should agree with those given in the steam tables.

Initial conditions: Point 1, Fig. 21:

$$p_1 = 30.0 \text{ lb per sq in. abs}$$
$$s_1 = 1.7201$$
$$v_1 = 14.392 \text{ cu ft per lb}$$
$$h_1 = 1178.9 \text{ Btu per lb}$$
$$t_1 = 280 \text{ F}$$

Assume an isentropic expansion to 23.0 lb per sq in., saturated, point 2, Fig. 21:

$$s_2 = 1.7204$$

Then

$$v_2 = 14.392\left(\frac{30}{23}\right)^{0.77} = 17.65 \text{ cu ft per lb}$$

Keenan's value for 23 lb (saturated) = 17.63 cu ft per lb

$$h_2 = 1178.9 - 0.804\,(30.0 \times 14.392 - 23 \times 17.65)$$
$$= 1178.9 - 20.58 = 1158.32 \text{ Btu per lb}$$

Keenan's value for 23 lb (saturated) = 1158.6 Btu per lb

$$T_2 = (280 + 459.6)\frac{23 \times 17.65}{30 \times 14.392} = 695.6$$
$$t_2 = 236.0 \text{ F}$$

Keenan's value = 235.49 F

For the entropy value $S = 1.7201$, condensation will occur at approximately 12.0 lb per sq in abs. The following example will serve to illustrate the loss due to supersaturation:

Let $p_3 = 12.0$ lb per sq in. abs, point 3, Fig. 21

$$s_3 = 1.7201$$

$$\text{Then } v_3 = 14.392\left(\frac{30}{12}\right)^{0.77} = 29.14 \text{ cu ft per lb}$$

$$\text{and } h_3 = 1178.9 - 0.804\,(30.0 \times 14.392 - 12 \times 29.14)$$
$$h_3 = 1113.0 \text{ Btu per lb}$$

From the Keenan steam tables and Mollier chart, for 12.0 lb per sq in. abs, wet steam, entropy of 1.7201,

$$h = 1111.39 \text{ Btu per lb}$$

The loss in enthalpy caused by supersaturation

$$\Delta h = (1113.0 - 1111.39) = 1.61 \text{ Btu per lb}$$

Since the enthalpy drop from the saturation line to the 12 lb pressure line along the entropy line, with s of 1.7201, is

$$H = (1158.6 - 1111.4) = 47.2 \text{ Btu per lb}$$

The loss due to supersaturation in this case is

$\frac{1.61}{47.2} = 3.41$ per cent of the isentropic enthalpy drop from the saturation line to the 12 lb wet-steam line.

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Discussion

A. G. Christie.[3] Mr. Yellott's paper forms a real contribution to our knowledge of the properties of steam. Much has been written upon supersaturation but this unstable state is so difficult to determine by ordinary instruments which measure temperature and pressure that all the material previously published has been in the form of deductions based on certain theories. Mr. Yellott's optical methods enabled him not only to observe the steam conditions in his nozzles but also to measure the droplet sizes and the pressures at which these formed. Greater reliance therefore can be placed upon these observations than upon any previous data.

It is a matter of regret that colored photographs of the actual jets could not be secured, particularly when polarized light was used. These would provide still more convincing evidence of the nature of the phenomena and the changes that actually take place in the flow of steam.

While Mr. Yellott's results have great value, many new problems arose in the course of his work. Another graduate student, J. T. Rettaliata, is continuing this research in an endeavor to determine whether roughness of the nozzle wall surface is the cause of the abrupt halt in pressure drop at the moment condensation starts from the supersaturated steam. This step in the pressure drop curves was noted by both Prof. Robb and Mr. Yellott.

An attempt will also be made to investigate the pressures at which condensation starts with varying nozzle shapes. Mr. Yellott found no preliminary condensation with nozzle No. 1 while with nozzle No. 2 preliminary condensation was usually present. We believe that a nozzle can be built between these two limiting sizes in which the condensation would be either ultimate or preliminary depending upon the pressure conditions. When this nozzle is developed and tested, its performance should give some clue to the determination of the conditions under which either form of condensation may be expected.

We also propose to try some actual turbine nozzles and to study both the supersaturation effects, the loss of contact of jets with walls, and the recompression in the issuing jet by means of light rays of definite wave-length.

It is undoubtedly a fact that this supersaturation phenomenon has an effect upon the expansion of steam in turbines at the saturation line and possibly even throughout the whole section of the turbine below saturation. In an earlier paper by Mr. Colburn and myself we suggested that supersaturation at the last blade row of the turbine may be an influencing factor in blade erosion. The influence of supersaturation on turbine performance with saturated or wet steam is another problem that has not yet been fully analyzed.

The paper before us will direct attention to these problems and as a result of further studies it is hoped that improvements in turbine performance may be effected.

J. Gershberg.[4] The writer would like to know if the author made experiments with a mixture of air and steam to bring out what influence, if any, air may have on supersaturation of steam.

This question is prompted by a rather peculiar phenomenon observed in a recent test of heat transfer in experimental condenser tubes. Two test tubes were placed side by side in the lower portion of the first pass of a two-pass condenser. One of these tubes was new and clean and the other old and dirty. The new tube was supplied with fresh water and the old one with salt water drawn from the house-service line. It was found that for a constant load on the turbine the water in the new tube was giving up instead of absorbing heat at the lowest water velocity employed in the test, this effect becoming progressively less as the water velocity was increased. However, the adjacent old test tube displayed a normal behavior, i.e., absorption of heat from the steam. Table 4 gives some of the test data at a water velocity of 2 ft per sec.

TABLE 4

Inlet water temp, F, condenser	46.3
Inlet water temp, F, new test tube	58.5
Inlet water temp, F, old test tube	55.6
Outlet water temp, F, new test tube	53.3
Outlet water temp, F, old test tube	59.0
Condenser hot-well temp, F	68.0
Abs pressure at turbine exhaust, in. Hg	1.13
Abs pressure at hot well, in. Hg	1.02
Air leakage, cu ft per min	7

It is to be noted that the two test tubes were located in proximity of the bank of tubes intended for cooling the air on its way to the air offtake.

Attempts were made to explain this phenomenon as due to partial pressures. However, computations do not seem to support this contention. They show that on the basis of partial pressures the lowest temperature of the fluid enveloping the new tube may be about 65 F which is close to 68 F in the hot well. The exterior fluid would be thus about 12 F hotter than the water at the outlet of the tube when, logically, it should be somewhat cooler. Also it was thought that, perhaps, a local cooling to this fluid was brought about by the 46.3 F inlet temperature of the main circulating water. Or is it possible that, notwithstanding the presence of water drops, a local supersaturation of vapor took place because of a comparatively richer mixture of air in that part of the condenser?

J. H. Keenan.[5] Supersaturation is a phenomenon which yields up its secrets only to the most persistent and intelligent observers. Mr. Yellott is one of these. He has presented data on supersaturation in an expanding stream that are more comprehensive and more illuminating than anything previously published.

It is well to recall that there are two classifications of supersaturated or undercooled steam. Supersaturated steam of the first class is any steam entirely free from water, but at a temperature lower than the steam table saturation temperature corresponding to its pressure. Supersaturated steam of the second class is steam in thermal equilibrium with small drops of water. This second kind of supersaturation, despite the presence of water drops, is undercooled relative to the saturation temperature given for its pressure in the steam table, because the steam table saturation temperature is correct only for an equilibrium mixture of steam with infinitely large drops of water. The first kind of supersaturated steam involves no liquid and its behavior is much like that of superheated steam. In fact, the best available method of determining its properties consists of extrapolating the constant temperature lines and other characteristics of superheated steam across the saturation line.

Mr. Yellott's photographs and the corresponding pressure curves show that supersaturation of the first kind exists in

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nozzle No. 1 down to a certain point at which two things occur: First, a dense fog of fine water particles is formed, not gradually but with great suddenness, and second, a step occurs in the pressure curve. It is reasonable to suppose that the steam carrying the fog is supersaturated in the second sense, but its degree of supersaturation depends upon the size of the fog particles.

The phenomenon of sudden condensation at a given section of the nozzle can be analyzed by means of the equations of continuity, energy, and momentum for a steadily flowing stream. Let p_0 and t_0 represent the pressure and temperature immediately before the nozzle where the velocity is negligible, and let sub-

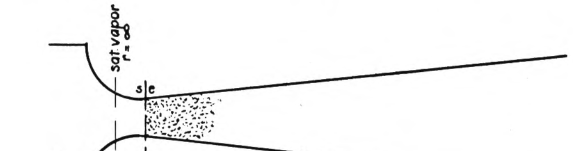

Fig. 22 Sample Pressure Distribution Curve
(The crossing of the normal saturation line and the condensation point are shown.)

scripts s and e designate properties of the steam at sections that are, respectively, immediately above and immediately below the condensation point, Fig. 22. Then the three equations may be written:

$$\text{Continuity, } V_s/v_s = V_e/v_e, \text{ since } a_s = a_e \quad [22]$$

$$\text{Energy, } h_s + V_s^2/2g = h_e + V_e^2/2g \quad [23]$$

$$\text{Momentum, } wV_s/g + p_s a = wV_e/g + p_e a \quad [24]$$

where V = stream velocity
v = specific volume
a = stream cross-section area
p = stream pressure
w = weight rate of flow
h = enthalpy

As expansion proceeds from p_o, t_o to p_s the steam is at first superheated but it becomes undercooled steam of the first kind once it has crossed the steam table saturation line. However, the change in name is not justified by any change in the characteristics of the expansion, because undercooled steam of the first kind has all the characteristics of superheated steam. Its name is changed only because a change in characteristics indicated by the steam table failed to materialize. Consequently, the velocity of the steam at p_s and its properties may be found by extending down to p_s the characteristics of the expansion from p_o, t_o to the steam table saturation line. Investigation shows that within the range covered by Mr. Yellott's tests isentropic expansion in the superheat region follows the polytropic

$$pv^{1.316} = \text{constant}$$

to a high degree of approximation. This relationship, combined with the properties of the steam approaching the nozzle and the measured pressure p_s, yields h_s, v_s, and V_s. Equations [22], [23], and [24] then become relationships between the four unknowns p_e, h_e, v_e, and V_e.

If we assume for the moment that the fog particles at e are infinitely large, that is, more than 1×10^{-5} in. in diameter, the ordinary table of the properties of saturated liquid and saturated vapor yields a fourth relationship between the four unknowns and offers the possibility of solution by a "cut and try" process. In a representative case for which p_o and t_o were 34 lb per sq in. and 297 F and p_s was 13.7 lb per sq in. (Fig. 22) the solution indicated a pressure at e 3.8 lb per sq in. higher than the pressure at s. In other words, to fulfil the requirements of the continuity, energy, and momentum equations, the pressure must rise suddenly when a sudden return to equilibrium occurs from the supersaturated state.

The step in the pressure curve at p_s serves to confirm this result. If a sudden pressure rise existed at this point, a search tube with a sampling hole of finite size would round off the discontinuity. Furthermore, since it is unlikely that all of the drops form at precisely the same instant, some additional flattening of the peak might be expected. However, an approximation to the pressure rise corresponding to instantaneous condensation may be found from the measured pressures by extending the trend of the curve following condensation back to the section at which condensation starts and measuring the difference between the extrapolated value and the measured value at that point. In the following paragraphs the pressure difference found in this fashion will be referred to as the measured pressure rise.

In the case under discussion the calculated pressure rise was roughly twice the measured rise, but the discrepancy might be explained by the small size of the drops. It seemed likely that if a solution of the analysis could be carried out for different drop sizes, each, of course, involving a different steam table relating p_e, h_e, and v_e for mixtures of that drop size with the correspondingly saturated steam, the drop size for which the calculated pressure rise agreed with the measured pressure rise would be, at least approximately, the size of the fog particles formed at s.

Consequently relationships between p_e, h_e, and v_e were found for equilibrium mixtures of vapor and water drops of different sizes. For this purpose the Kelvin-Helmholtz equation for the supersaturation ratio, the Stodola equation for capillary energy of the drops and the polytropic exponent 1.316 for steam at temperatures above the corresponding equilibrium temperature were employed.

The condensation pressure rise for a number of different drop sizes was then computed and plotted in Fig. 23. It should be noted that the drop size corresponding to zero pressure rise upon condensation is the size given by Mr. Yellott. That is, Mr. Yellott calculated the drop size starting from the assumption that condensation begins as soon as the expanding steam crosses the saturated vapor line corresponding to the size of the drop formed. In this he was following the precedent set by authoritative earlier investigators. That condensation does not begin, however, as soon as the expanding steam crosses the saturated vapor line corresponding to the size of the drops about to be formed is indicated by Mr. Yellott's own experimental results as follows:

(*a*) His photographs of nozzle No. 1 do not indicate a gradual increase in the number of water particles starting from a trace at the saturation point and increasing in numbers to a thick fog further on. On the contrary, a thick fog is formed suddenly as the steam crosses a definite section. It is true that traces of liquid appear in nozzle No. 2 before the thick fog is formed, but even here the major condensation occurs suddenly

(*b*) The step in the curve of pressure through the nozzle is an indication of a discontinuity in the sequence of state changes through the nozzle. Had condensation occurred at the proper saturation line no more discontinuity would have been observed than would be calculated from the ordinary steam table for isentropic expansion into the moisture region.

The measured pressure rise for an initial pressure of 34 lb per sq in. was about 1.8 lb per sq in. which, according to Fig. 23, corresponds to a drop radius of about 5×10^{-9} ft which is about two and one-half times the radius, or about fifteen times the volume, calculated by Mr. Yellott.

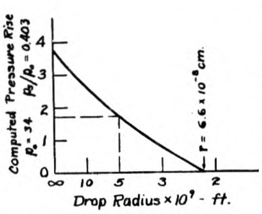

FIG. 23 PRESSURE RISE FOR SUDDEN CONDENSATION FROM SUPERSATURATED STEAM, PLOTTED AGAINST RADIUS OF DROPS FORMED

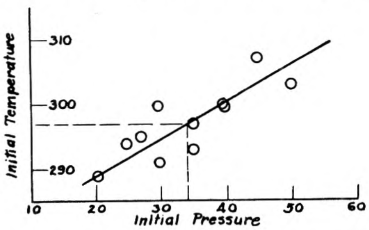

FIG. 24 INITIAL TEMPERATURE OF THE EXPANDING STEAM

Whether this same size drop was formed in all of the tests on nozzle No. 1 could be learned by comparing calculated pressure rise with measured pressure rise in each instance. To simplify the calculations initial temperatures were taken from the curve of Fig. 24 and p_s was taken from the curve of Fig. 25. In both instances the test values, shown as circles, departed little from the curve assumed. Two tests were omitted from the analysis, though they conflict in no way with its results, because their initial temperatures were out of line with those of the other tests.

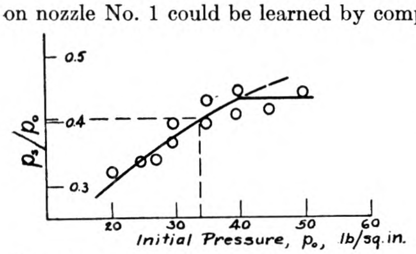

FIG. 25 CONDENSATION PRESSURES FOR VARIOUS INITIAL PRESSURES
(Circles are measured pressures.)

The measured pressure-rise values are shown as circles in Fig. 26. They were obtained before the analytical values were computed in order to avoid, as far as possible, the influence of personal prejudice.

The condensation pressures, p_s, plotted in Fig. 25 show a consistent trend for initial pressures between 20 and 40 lb per sq in. Two tests at higher pressures failed to follow that trend. It seemed best to extrapolate the trend to obtain condensation pressures for computation purposes until it was found that there was no corresponding solution for an initial pressure of 50 lb per sq in. It became evident that the requirements of continuity, energy, and momentum cannot be satisfied for this drop size and for the extrapolated condensation pressure. But solutions were found at the higher pressure when values of p_s were chosen from the solid line of Fig. 25 which follows the measured values. These solutions give the solid line of Fig. 26.

The agreement between the circles and the solid curve of Fig. 26 is evidence of the validity of the analysis. The discontinuity in the trend of the condensation pressure data proves, quite unexpectedly, to be further evidence in its favor.

It appears, then, that the expansion of steam in a nozzle into the region below the normal saturation line involves complete supersaturation until a vapor state is reached which would exist in stable equilibrium with droplets of about 2×10^{-9} ft radius (6.5×10^{-8} cm), but which is undercooled compared with steam in equilibrium with any larger size drops. From this state some of the steam suddenly condenses to form a fog of droplets 5×10^{-9} ft radius and releases energy sufficient to bring the surrounding vapor into substantial equilibrium with these relatively large drops. This process is essentially irreversible as indicated by the suddenness with which it occurs, but gradual condensation to drops of 2×10^{-9} ft radius would have been reversible except for friction between vapor and droplets. Irreversibility does not mean that the drops cannot be reevaporated upon compression of the vapor carrying them. It means only that some "hysteresis" effect must accompany such reevaporation. Thus, a vapor which expands to a state undercooled relative to the size of drop subsequently formed must be compressed after condensation to a temperature markedly higher than that at which condensation occurred before all drops are evaporated again.

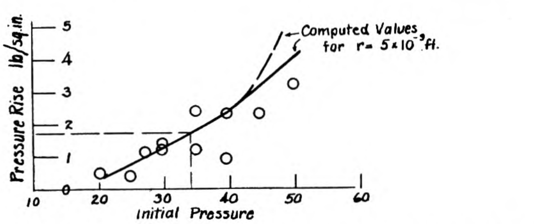

FIG. 26 MEASURED PRESSURE RISE COMPARED WITH CALCULATED PRESSURE RISE FOR FORMATION OF DROPS 5×10^{-9} FT RADIUS
(Circles are values obtained from test curves. The solid line is calculated using values from the solid line of Fig. 25. The broken line is based on the broken line of Fig. 24 for which solution became impossible between 45 and 50 lb per sq in. initial pressure.)

The discontinuity in the course of the pressure which characterizes the condensation point is doubtless evidence of further irreversibility. In fact it is not very different in nature from the shock effects occurring in the diverging portions of nozzles discharging against a back pressure higher than the throat pressure. It results in a kinetic energy loss in excess of the amount usually calculated for supersaturation by assuming that condensation proceeds, however irreversibly, at constant pressure. Table 5 compares the kinetic energy in a stream which expands from 34 lb per sq in. 297 F to various pressures for

(*a*) isentropic expansion in equilibrium with infinitely large drops for all pressures below the normal saturation line

(*b*) isentropic expansion without moisture ($pv^{1.316}$ = constant), and

(*c*) isentropic expansion without moisture to a pressure below the final pressure followed by sudden compression to the final pressure simultaneous with the formation of infinitely large drops.

The difference between (*a*) and (*b*) is what has hitherto been called the supersaturation loss. The difference between (*a*) and (*c*) is a somewhat larger value by virtue of the shock effect and is doubtless more nearly correct for this case. The differences between the losses calculated by the two methods are rather small and somewhat irregular, perhaps from slight steam table irregularities. They indicate, however, that the shock effect increases the loss by about $1^1/_4$ per cent of the kinetic energy if condensation forms as very large drops. Computations have not been made for drops 5×10^{-9} ft in radius but it is likely that they would show an additional loss of about $^5/_8$ per cent instead of $1^1/_4$ per cent.

It is of some importance to know whether condensation occurs in the steam stream of a turbine in the constant pressure fashion or in the shock fashion. In general it may be said that condensation will occur where the pressure of the stream is falling since once a stream reaches a given pressure without condensation no continuance at that pressure will result in condensation (except on cold walls), particularly if the temperature of the stream is rising because of friction. A stream which is falling in pressure along its direction of motion is generally constrained by walls or by dynamic limitations to a certain sequence of cross-section areas and under such conditions the equation of continuity will doubtless be valid when applied to the two sides of the condensation section. Consequently, it is logical to assume that this shock type of condensation is the important type to the turbine designer and it will justify further study.

TABLE 5 KINETIC ENERGIES AND SUPERSATURATION LOSSES FOR EXPANSION FROM 34 LB PER SQ IN. AND 297 F

(Condensation on infinitely large drops)

	Kinetic energy at final pressure Btu/lb			Loss in kinetic energy			
Final pressure, lb per sq in.	*a* Equilibrium with large drops	*b* Supersaturation constant *p* condensation	*c* Supersaturation shock condensation	Const. *p* condensation *a* — *b* Btu/lb	%	Shock condensation *a* — *c* Btu/lb	%
6.89	114.4	108.4	106.6	6.0	5.25	7.8	6.8
12.63	73.7	72.0	71.2	1.7	2.3	2.5	3.4
17.51	50.9	50.2	49.6	0.7	1.3	1.3	2.6

Mr. Yellott should be strongly urged to investigate further these supersaturation and moisture region phenomena about which much has been said and little has been known. It is of great importance to find out what happens in stages succeeding the initial condensation stage. Does supersaturation persist in each succeeding nozzle, or does the presence of the original fog inhibit it? On the answer to this question hinges our further understanding of moisture region phenomena in the steam turbine. Various authorities on steam turbine design have agreed that the loss due to moisture and supersaturation combined, affects the efficiency of a turbine stage slightly more than 1 per cent for each per cent of moisture present. One authority[6] has attributed practically all of this effect to supersaturation. Others[7] show that the entire moisture region loss might be readily accounted for by friction between steam and water particles.

Would it not be possible to feed the glass walled nozzle with steam from a turbine stage? A suitable moisture content might be found by tapping the proper stage of a multistage turbine. Large water drops swept from metal surfaces could be largely eliminated by a centrifugal separator leaving only the fine fog within the stream. Then, photographs and pressure tube explorations might tell something of how condensation progresses in these later stages, whether continuously or by shock effects. Something might be learned concerning the relative distribution of water between the minute fog particles, which probably result in but small friction loss between steam and particle, and the relatively enormous drops which are swept from surfaces and which require large friction forces as their speeds are alternately increased and decreased down through the turbine.

[6] H. M. Martin, *Engineering*, vol. 106, p. 1, 1918.

[7] G. A. Goodenough, *Power*, vol. 66, p. 466, 1927.

F. W. Gardner.[8] Mr. Yellott's results seem to give a very satisfactory confirmation of the work of others who have carried out research in the same field, and the existence of supersaturation in a simple nozzle seems now to be well established. Our difficulty in actual turbine work is that we have never been able to obtain any satisfactory evidence that it occurs in turbine blades, or at all events, that it has any appreciable effect on the turbine performance.

It is questionable whether the drop size, as deduced from the saturation experiments, has an important influence on the problem of blade erosion. The drops that produce erosion on the exhaust end blades of an actual turbine are probably of a much greater order of magnitude and result from the accumulation of water on the preceding blade surfaces. The size of drop that can persist under these conditions is discussed by Soderberg, and in an ordinary turbine is probably of the order of a hundredth of a centimeter in diameter, as compared to the figure of 6×10^{-5} centimeters given by Mr. Yellott.

In connection with the effect of ionization on condensation, Sir Charles Parsons and Mr. Dowson carried out some experiments a few years ago. Following some preliminary experiments in which a high-tension discharge was observed to produce a marked effect on the steam cloud issuing from an open ended nozzle, an apparatus was developed for ionizing the steam supply to a small turbine driving a dynamo. In this experiment voltages up to 100,000 were employed and tests were made to see if any difference in consumption, speed, or output could be detected. These experiments, however, gave a negative result, and the performance of the unit appeared to be quite unaffected by the introduction of the high-tension discharge.

Author's Closure

The new problems mentioned by Professor Christie have been the subject of further research carried on by Mr. Rettaliata at the Johns Hopkins University and by the author at the University of Rochester. It is hoped that the results of this work may be presented at a subsequent meeting of the Society, and that colored motion pictures will be available to illustrate the optical features.

In response to Mr. Gershberg's question, the presence of air in steam cannot be detected by this method, for the air is quite as transparent as the steam. It is probable that the phenomena which he noted are due in some manner to the low temperature of the main cooling water rather than to supersaturation.

The author wishes to express his gratitude to Professor Keenan for his excellent analysis, which not only gives encouraging support to the accuracy of the experimental results, but also provides a new theoretical method of attack upon the condensation problem. The only amendment to his method which the author can suggest is a slight modification of the Stodola equation for the capillary energy of droplets. Introduction of an improved equation for surface-tension results in values slightly greater than those calculated from Stodola's equation.

The experimental work which the author has been conducting with improved apparatus during the past year at the University of Rochester may supply the answer to some of the questions raised by Professor Keenan. A number of interesting and perhaps significant facts have been observed, and an attempt is being made to deduce the general laws which control the condensation of flowing steam.

[8] C. A. Parsons and Co., Ltd., Heaton Works, Newcastle-on-Tyne, England.