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INTERCHANGEABLE INVOLUTE GEARING

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Member of the Society

39 This paper was prepared at the request of the Committee on Meetings of The American Society of Mechanical Engineers as the contribution to the discussion upon the need of a standard or standards for gears to be considered at the meeting with The Institution of Mechanical Engineers in July 1910. By way of explanation, it may be said that its author is chairman of a committee on Standards for Involute Gears, appointed about a year ago by the president of The American Society of Mechanical Engineers to investigate the subject of interchangeable involute gearing, and to recommend, if found desirable, a standard or standards. This paper, however, is not to be considered as an expression of the opinions of the other members of the committee, except where so stated.

40 After more or less unsatisfactory experience with cycloidal gearing, I investigated about twenty-five years ago the subject of involute gearing with the object of determining upon a system, for the firm of Wm. Sellers & Company, with which I was then connected. The conditions imposed called for a system applicable to any number of teeth between a 12-tooth pinion and a rack, without change in the Sellers addendum which had always been made 0.3-pitch for the cycloidal teeth, hitherto used almost exclusively by them.

41 I found that the involute forms then in vogue were confined to obliquities of 14½ deg. and 15 deg. with an addendum equal to the modulus, or about 0.32-pitch. This long addendum with such small obliquities naturally gave rise to interference between racks and pinions of less than 30 teeth, and rather than modify the involute form I finally recommended the adoption of a pressure angle of 20 deg. At the same time, I was well aware of the fact that even this obliquity was not sufficient to prevent interference between a 12-

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toothed pinion and a rack, but for such pinions and the gears of 60 teeth or less, with which they commonly engaged, I believed the interference would not be noticeable in practice. I was strongly tempted to go further and fix upon an obliquity of $22\frac{1}{2}$ deg., but 20 deg. then appeared such a radical departure from common usage that the advantages of the greater angle were dismissed as being possibly more theoretical than real.

42 The 20-deg. system with an addendum of 0.3-pitch has now been in use by Wm. Sellers & Company for twenty-five years and has given satisfaction in a general way, although the interference referred to has been more or less noticeable on 12-toothed pinions. I reviewed this matter ten years ago in a paper read before the Engineers' Club of Philadelphia, advocating an obliquity of $22\frac{1}{2}$ deg. and suggesting as a much-needed reform in engineering practice the consideration of uniformity in interchangeable gearing. I then pointed to the action of the Franklin Institute more than thirty years earlier, which inaugurated a standard system of screw threads and expressed the hope that by the interchange of opinions an agreement among engineers might be reached leading to the gradual disappearance of needless diversity in the forms of gear teeth.

43 Nothing in this direction had been done however, when the subject of interchangeable involute gear-tooth systems was brought to the attention of The American Society of Mechanical Engineers in a paper by Ralph E. Flanders presented in December 1908. A number of systems in general use were analyzed and their merits discussed from various points of view and the desire expressed that the Council be petitioned to appoint a committee to investigate the subject of interchangeable involute gearing and, if found desirable, to recommend a standard or standards.

44 In answer to this petition the Council voted in January 1909, that the President appoint a committee of five members to formulate standards for involute gears and present the same to the Council. Without anticipating in any way the conclusions of this committee yet to be formulated, if indeed an agreement be possible, I believe it will be helpful to give publicity to the line of investigation upon which we have embarked and thus obtain the benefit of such criticism or encouragement as it may provoke.

45 At the first meeting of the committee in June 1909, it was decided to obtain an expression of opinion from the manufacturers of gears and gear cutters and later in October the following circular letter was sent out:

Dear Sir:

A committee of The American Society of Mechanical Engineers has been appointed to consider the subject of standards for interchangeable involute gears, and would like to have as much light upon the subject as can be given them by the manufacturers.

Without limiting in any way the type of gears to be used for any special service, it is believed to be desirable that some type should be known as standard so that any gears of that standard may run together and be perfectly interchangeable.

The committee is, therefore, interested to know the extent to which involute gears are modified in your practice to avoid interference, and it would be pleased to have your ideas as to what, in your opinion, should constitute standard interchangeable involute gears. Any suggestion you may be pleased to offer will be very much appreciated.

Yours truly,

WILFRED LEWIS, *Chairman*
HUGO BILGRAM
E. R. FELLOWS
C. R. GABRIEL
GAETANO LANZA

Committee on Standards for Involute Gears

46 In response to the circular letter above referred to, about one hundred answers were received, expressing more or less interest in the subject and giving more or less conflicting preferences and conclusions.

47 One correspondent said he would hail with delight any system whereby complete interchangeability could be attained on gears running without noise up to 1000 ft. a minute, as among automobile manufacturers noiseless gearing was always the chief end in view.

48 It is undoubtedly true, as pointed out by other correspondents, that perfect cutters do not necessarily produce perfect gears and that close attention must be paid to the setting of the cutter, proper indexing, speed, feed, etc., and from the extent to which the present 14½-deg. system is established, it has been argued that even if the pressure angle of 14½-deg. were not the most desirable, it would be better for the sake of the established practice to let it alone. Quite a number share this opinion and argue against the recommendation of a standard which may simply add another system to those already in use.

49 It may be said in reply that the chief advantages of involute gearing over cycloidal gearing which it has pretty generally displaced are the comparative simplicity of the involute curve and a slight variation in center distances not permissible in any other system of gearing. Despite the difficulty of securing general recognition for

any system of interchangeable gearing, I believe the committee should investigate and report upon some ideal standard or standards.

50 Attention is called by a number of correspondents to the increasing number of gears cut by the hobbing process and the difficulty of making such gears interchangeable with certain cut gears, thus indicating the hobbing principle as the first principle to which all cutters should conform, the rack being the basis of all generated gears. It is admitted that hobbed gears can be used interchangeably with gears cut by certain other processes, but some doubt is expressed of the possibility of realizing an interchangeable system using pinions with a small number of teeth and teeth of reasonable length without resorting to some modification of the involute curve. The opinion is freely expressed that if such modification becomes necessary the method should be known and clearly defined, so that makers of gears can adopt it and, if necessary, make their own cutters.

51 As pointed out in Mr. Flanders' paper and as mentioned repeatedly by our correspondents, the most desirable quality in gearing and the one by which it is almost universally judged, is quietness and smoothness of running. Next to this come strength, durability and permanence of form, and upon the last, of course, depend continued quietness and smoothness of action. Friction and journal pressure are of less importance, but still worth considering, and before reaching any conclusions from theoretical considerations alone, we propose to determine if possible, in a practical way, the relative advantages of some of the systems in common use and, with these, other systems to which we are disposed to give favorable consideration.

52 It is understood that the Institution would be pleased to have the coöperation of the Society in the work they have in hand looking to a determination of the friction in the transmission of power by gearing. In this connection we have been reminded of a suggestion of Prof. J. Burkitt Webb of Stevens Institute of Technology made soon after the publication of the Sellers' experiments upon gears reported to our Society in 1885. In these experiments an attempt was made to measure the friction loss between the teeth of a pair of spur gears, but the apparatus used, made originally for testing the friction of worm gearing, was not delicate enough for the purpose and the errors introduced exceeded, in some cases, the net result.

53 Professor Webb then suggested the possibility of so dividing one of the pair of spur gears to be tested as to make the load on the

teeth self-contained. The apparatus which we have designed embodies this idea, thus making it possible to run gears under heavy loads at high speeds with a very small consumption of power. We have also provided in our apparatus for an adjustment of center distance and means to measure the thrust between centers while the gears are running. Of course, the thrust between centers can be estimated very closely for involute gears from the pressure angle on the teeth, but we anticipate results somewhat in excess of this on account of the excess in friction of approach over that of recess, and, if any but involute gears are tested, it will also be interesting to compute from experimental data the effective obliquities of other systems.

54 We propose to determine the friction loss under various speeds and pressures for wheels and pinions cut to the Brown and Sharpe 14½-deg. standard, the 20-deg. stub tooth and a 22½-deg. tooth with addendum of $\frac{7}{8}$ module or about 0.278 pitch. These gears will be tested at normal center distance, and also at distances about 1 per cent or 2 per cent of the pitch greater or less than this, and an effort will be made to record graphically the noise produced under these different conditions.

55 We believe that accuracy and permanence of form can thus be given their proper influence on the reduction of noise. It may take some time to determine the effect of wear, but from the method of loading the teeth and the small amount of power consumed some indication of the tendency of wear can be obtained. All gears tend to wear out of shape, and involute gears more so than cycloidal, but we recognize as a possibility that this tendency may be checked by the deformation itself and also that the loss in friction at different parts of a gear tooth is practically incalculable on account of the variations in friction for different velocities of sliding.

56 The experiments we propose should therefore give information unobtainable in any other way and throw a flood of light on the problem in hand.

57 As it is quite impossible for any of the committee, who are all busy men, to make the experiments here outlined, we have been fortunate enough to interest, through the intervention of Professor Lanza of the Massachusetts Institute of Technology, two of his undergraduates, Messrs. Green and Doble, in making these experiments the subject matter for a graduating thesis. Professor Lanza hoped to have the work completed in June so that the results might be communicated for discussion at our meeting in England in July but this has proved impossible, partly because of the magnitude of the under-

taking and partly of the delay in the completion of the testing machine.

58 The apparatus used in making these experiments is shown in the photographs, Fig. 17 and Fig. 18, and the line drawing, Fig. 19, which gives some of the principal dimensions and shows the knife edges on which the machine rests. The machine consists of a frame *A* designed to carry a pinion shaft in roller bearings at one end, and a frame *B* pivoted to it and designed to carry the gear wheels *W* engaging with a wide-faced pinion *P* on the pinion shaft. The frame *B* is held to the

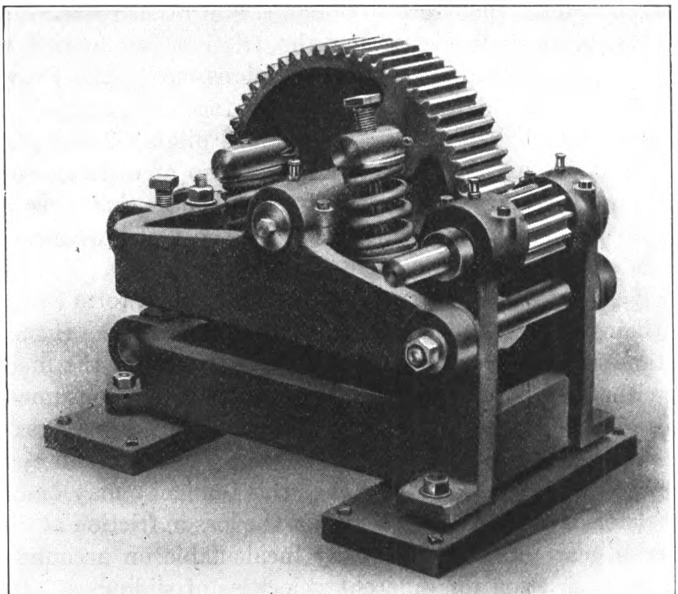

FIG. 17 GEAR TESTING MACHINE

frame *A* at its outer end by an adjustable clamping bolt *C*, and provision is made to measure the thrust on centers by means of the spring *D* acting between the frame *A* and an adjustable abutment on the frame *B*. The gear wheels to be tested consist of a central gear with a wide face and two side gears with narrow faces. The central gear carries two heavy cross pins *G*, which pass through clearance holes in the side gears, and the side gears carry two heavy pins *H*, which pass through clearance holes in the central gear. Between the projecting ends of these pins G and H, heavy helical springs S are inserted, upon which pressure can be applied by means of the set screws K.

59 The pressure of these four springs S is resisted by the gear teeth, the middle gear pressing against one side of the pinion teeth and the side gears pressing against the other side. The pinion thus becomes simultaneously a driver and a driven gear and the power required to turn it when loaded in this way is only that required to overcome the friction of the teeth and whatever resistance there may be in the gear journals. The latter presumably is very small

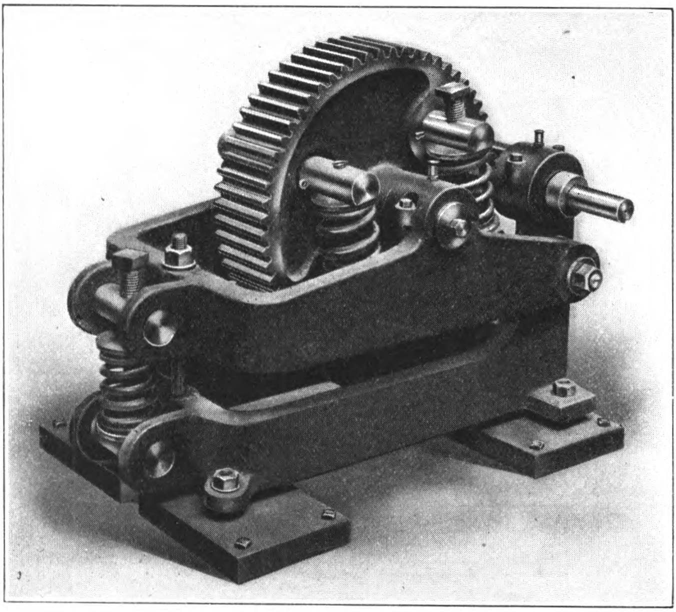

FIG. 18 VIEW SHOWING ARRANGEMENT OF PIVOTED FRAME AND SPRING FOR MEASURING THRUST

indeed, but provision has been made to measure it by substituting plain cylinders without teeth for the gears and pinion, and running these under the same journal pressures. By deducting the resistance due to journals from the total resistance with running gears, the friction of the teeth alone can be determined.

60 In operation this machine is driven by an extension to the pinion shaft, carried to bearings several feet distant to permit of ample flexibility. The knife edge directly beneath the pinion rests

upon a permanent support, and the other knife edge is carried upon the platform of a small platform scales. The driving moment in the pinion shaft will therefore be measured by the scale reading at the end of an arm 20 in. long, and by reversing the direction of motion given to the pinion shaft the effect of any initial lateral strain in the driving shaft can be eliminated.

61 Fig. 20 illustrates three types of gearing to be tested by Messrs. Green and Doble, and with these other types may be included later on.

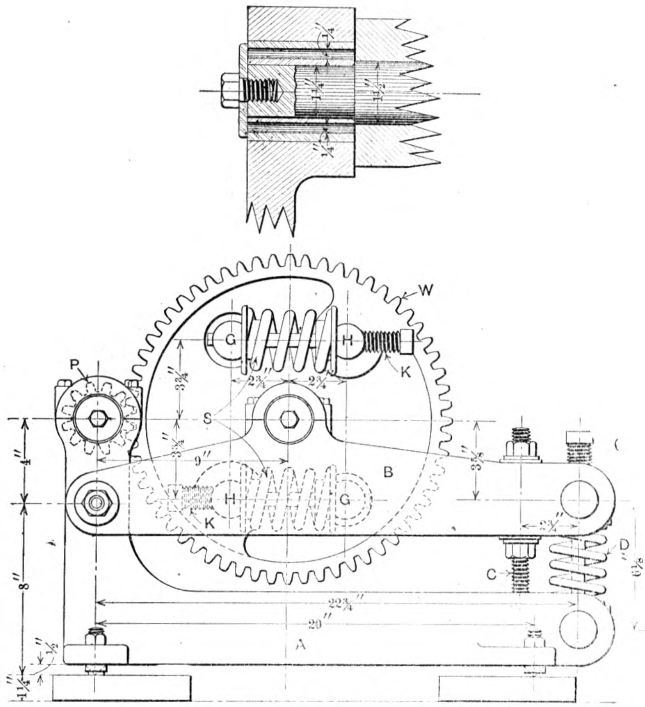

FIG. 19 DIAGRAM OF GEAR TESTING MACHINE

62 Figs. 21, 22 and 23 illustrate a group of involute gears designed by Mr. Bilgram to engage a rack of 15-deg. obliquity, and to demonstrate the possibility of using pinions of ten or even nine teeth with such a rack, provided the addendum can be varied. Without wishing to advocate the use of a variable addendum in interchangeable gearing, it is interesting to note the possibility of making a tentative solution of the problem in this way. A set of these gears has kindly been

furnished by Mr. Bilgram for making a comparative test. He has also made a set of models, from which the figures have been photographed, and referring to them, he gives the following explanation:

63 "While the involute system of gearing has decided advantages over any other, it has the one disadvantage that the faces of the

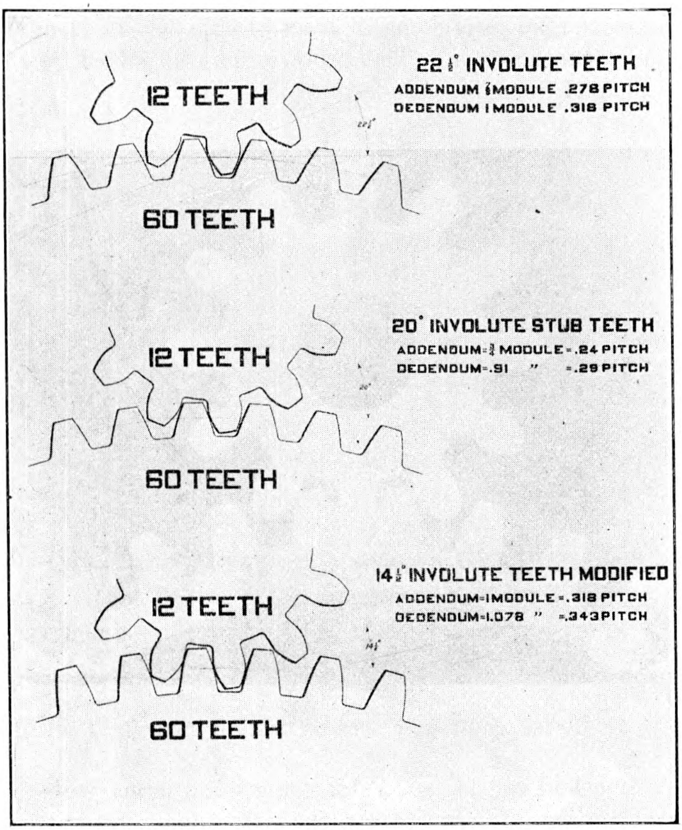

Fig. 20 Gear Types Tested in the Machine

teeth of wheels come into interference with the flanks of pinions, if the latter have a comparatively small number of teeth. Unless the flanks of the latter are undercut, the teeth will interlock or at least mesh improperly.

64 "In making a single pair of wheels, a remedy can readily be applied. There are two ways in which interference can be avoided,

namely either by increasing the angle of pressure or by shortening the addendum of the wheel. If the latter method is chosen and it is desired not to reduce the working depth of the teeth, it is necessary to add to the addendum of the pinion the amount taken from the addendum of the wheel.

65 "This latter method is out of the question when the problem is given to make an interchangeable set of spur wheels from a rack down to a 12-tooth pinion. This problem may be solved by a combination of both remedies alluded to."

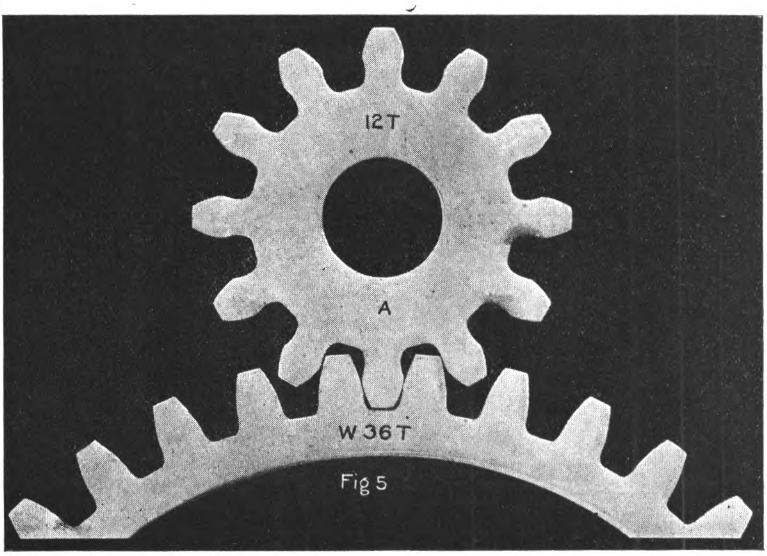

FIG. 21 BILGRAM SYSTEM: PINION WITH UNDERCUT TEETH

66 The method consists of making racks and larger wheels with normal addendum, but increasing the addendum of pinions just enough to prevent the rack tooth from interfering with the flank. The samples presented (Figs. 21 to 23) consist of a rack R and a 36-tooth wheel W, with angle of pressure of 15 deg. and addendum equal to the modulus. The 12-tooth pinion A, generated by a rack, corresponding to rack R, shows the undercutting thereby produced. Obviously this pinion will not work, as so much of the involute is cut away that the path of contact is materially less than one pitch. But there are also shown pinions of twelve, ten and nine teeth, made with increased addenda. These were generated by a rack like R,

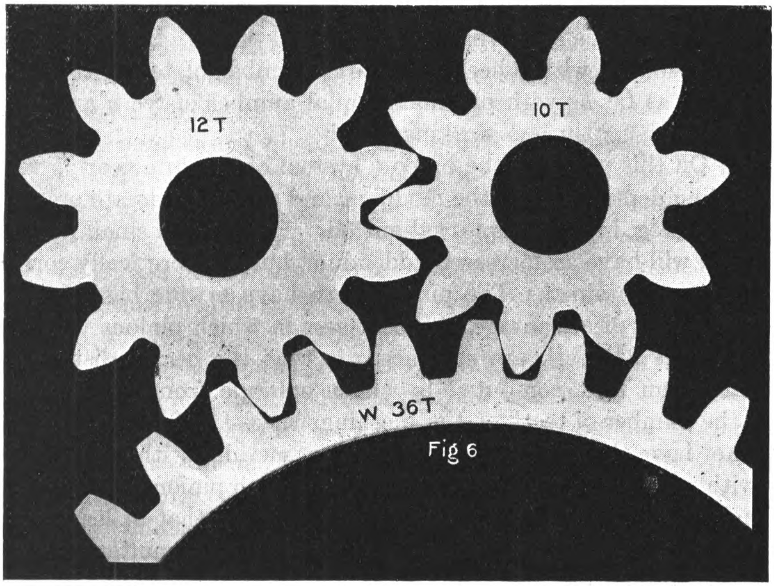

FIG. 22 BILGRAM SYSTEM: PINIONS WITH INCREASED ADDENDA IN MESH WITH GEAR

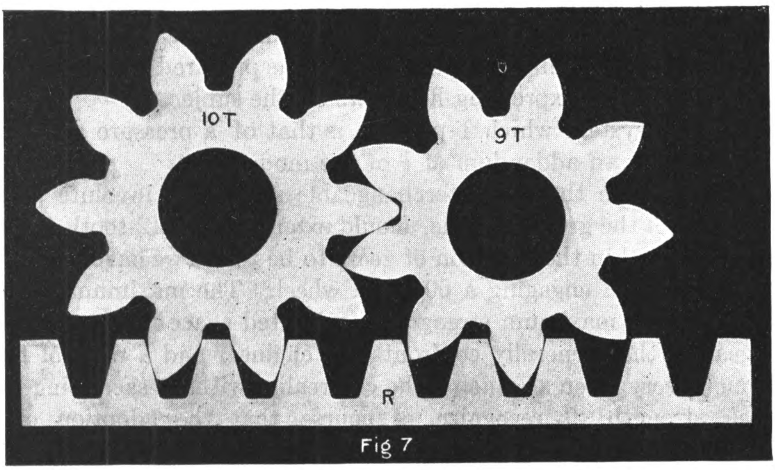

FIG. 23 BILGRAM SYSTEM: PINIONS WITH INCREASED ADDENDA IN MESH WITH RACK

but with a somewhat greater addendum than that used in generating the wheel *W* and a somewhat greater cutting depth. If these pinions are then mated with wheels of a large number of teeth, they will not enter as far as with pinions of equal number of teeth and have therefore a slightly less working depth.

67 On this plan may be based a system of involute gearing with a working depth of twice the modulus, and with a moderate pressure angle, 15 deg. in the samples submitted. Pinions of a small number of teeth will have an increased addendum, but a theoretically correct action is maintained. The pinion teeth have a wide base and are strong. One disadvantage in those cases in which pinions with less than about 24 teeth are embraced, is that the center distance is greater than that computed by the usual rule from the modulus and the number of teeth of the meshing wheels. Moreover, pinions will not have the full working depth when meshing with large wheels or with racks, but even in the case of a 10-tooth pinion meshing with a rack, the path of contact exceeds one pitch so that at least for a portion of the action two teeth will be in contact simultaneously.

68 The plan proposed by Mr. Fellows is to use an involute with an angle of pressure of 20 deg. and to reduce the addendum to ¾ of the modulus. Such teeth are known as "stub teeth". By this method interference in case of a rack gearing with a 12-tooth pinion is just avoided and in the case of two 12-tooth pinions meshing with each other the path of contact is equal to about 1⅛ of the pitch.

69 Mr. Gabriel prefers the 14½-deg. standard of the Brown and Sharpe Manufacturing Company and he has prepared a contribution to the discussion expressing his views on the subject.

70 The system which I propose is that of a pressure angle of 22½ deg. and an addendum of ⅞ of the modulus.

71 I believe that an interchangeable system of involute gearing, to be of the greatest value, should extend from a 12-tooth pinion to a rack, and in the selection of gears to be tested we have chosen a 12-tooth pinion engaging a 60-tooth wheel. The maximum reduction with the maximum strength in a limited space is the problem in gearing that generally confronts the engineer and a ratio of five to one is very often as much as he can realize without sacrificing too much strength. I recognize, of course, that the adoption of a larger number of teeth in the smallest allowable pinion overcomes some difficulties, and that this may be a debatable point, but I do not think any system of interchangeable gearing will be satisfactory which does not include pinions of twelve teeth. The cycloidal

system formerly employed was based upon a 12-tooth pinion, and I believe this number can be retained for the smallest number in an interchangeable involute system without serious objection. That is, there is better ground for the retention of this minimum number than for a higher number and when the pros and cons have all been summed up there will be no change in this well-established minimum for interchangeable pinions.

72 Although the experiments made under the direction of Professor Lanza are not by any means conclusive, enough has been done to indicate that the friction loss in gear teeth is influenced to a greater extent by the length of the addendum than by the obliquity of the system. Theoretically, the friction loss in involute teeth is independent of the obliquity and increases with the addendum. The loss in journal friction should vary as the secant of the pressure angle, but the latter is also affected by the dead weight on journals which even with plain bearings is a small matter, while with ball or roller bearings, it is a very trifling consideration indeed.

73 I believe therefore that a pressure angle of $22\frac{1}{2}$ deg. can be adopted without fear of reduced efficiency in the transmission of power; that an addendum of $\frac{7}{8}$ module will give an ample arc of action for all combinations of gears between a 12-toothed pinion and a rack, and that true involute forms made to these constants will avoid the necessity for any empirical modifications and give results comparable with the best now obtained by such means.