No. 1865

THE DESIGN OF COOLING TOWERS[1]

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The author points out that engineers have designed cooling towers in the past on empirical information and in accordance with the experience of previous successful designs. Wide departure from standard designs is difficult because of the lack of scientific basis for the design. The author therefore establishes the general principle applicable to cooling-tower design and derives equations for the use of the designer. He presents a quantity of experimental data to substantiate the validity of his formulas and shows by an actual experiment how these formulas are applicable to the design of a counter-current cooling tower.

THE factors influencing the design of cooling towers have been studied by a number of investigators. There has been, however, no statement of the influence of these factors of such a nature that it has been possible to calculate easily the performance of a given tower under widely varying internal and external conditions. Engineers have found that towers of specified construction, when operated at specified air and water rates, may be expected to cool water to within a certain number of degrees of entering air temperature and to discharge the air from the tower nearly saturated at some temperature approaching that of the entering water. In the absence of anything but empirical information, towers are therefore constructed along certain well-established and successful lines. Thus the designer of the forced-draft tower allows the water to fall through a height of perhaps 22 ft. and proportions the ground area on the basis of 6½ gal. per sq. ft. of ground area per min. However, this empirical knowledge will not enable the engineer to predict what will happen when conditions depart widely from standard practice, as, for instance, the capacity of a tower 11 ft. or 44 ft. high. Nor could he estimate the tower height required to cool a given volume of

[1] This paper is based upon the principles demonstrated by Prof. W. K. Lewis, see paper No. 1849 of this volume of Trans., and is an application of these principles to one particular field.

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air from Death Valley at 120 deg. fahr. and 4 per cent relative humidity to 70 deg. fahr. by means of a parallel-current tower.

2 Investigations carried on by the Department of Chemical Engineering at the Massachusetts Institute of Technology under the direction of W. K. Lewis in connection with humidification and air drying, have led to the development of fundamental conceptions as to the mechanism involved in the transfer of heat between liquids and gases and in the vaporization and condensation of liquids and vapors. It is the purpose of this paper to show how these concepts can be applied to the particular case of cooling towers, and to devise by these means methods by which the engineer can simplify their design.

GENERAL PRINCIPLES

3 There are two principles upon which all of the subsequent work will be based, (*a*) the conservation of matter and energy, and (*b*) the potential concept. The latter may be expressed briefly as the effect upon the rate of flow of matter or energy of the driving force applied.

4 *The conservation of energy* as applied to a cooling tower may best be shown by means of a heat balance. This can be written as —

$$s_w w(t_1 - t_0) = Ws'(T_1 - T_0) + Wr'(H_1 - H_0) \quad . \quad . \quad . \quad [1]$$

where s_w = average specific heat of the water between the bottom and top of the tower

w = weight of water leaving the tower

s' = humid heat [1] of the air entering the tower, that is, the heat required to raise one degree in temperature one pound of dry air plus the water vapor H that it contains

T = temperature of air

t = temperature of water

W = weight of air (moisture free) entering tower

r' = total heat of water vapor at the top of the tower at the temperature of the leaving air minus the heat of the liquid of water at the temperature of the entering water.

The subscripts 1 and 0 refer to the top and bottom of the tower, respectively.

[1] Wm. S. Grosvenor, Trans. Am. Inst. Chem. Engrs., 1908.

5 Equation [1] is used universally by engineers at the present time in cooling-tower problems, usually in the approximate form —

$$w(t_1 - t_0) = Ws(T_1 - T_0) + Wr(H_1 - H_0) \quad . \quad . \quad . \quad . \quad [2]$$

where s and r are the average humid heat and latent heat between the top and bottom of the tower, respectively.

6 *The potential concept* may be applied to both the rate of transfer of heat from the liquid to the gas and to the rate of diffusion of water vapor from the liquid to the gas, as shown in the following paragraphs.

7 The rate or flow of heat is proportional to the temperature difference between the liquid and the gas, and the rate of diffusion of water vapor is proportional to the difference between the vapor pressure of the liquid water and the partial pressure of the water vapor in the gas.

8 The amount of heat flowing from the water to the gas per unit time would therefore be —

$$WsdT = haAdx(T - t) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [3]$$

where h is the coefficient of heat transfer per unit area (sq. ft.), a the square feet of cooling surface per cubic foot of volume of the tower, A the cross-sectional area of the tower, and x the height of the tower.

9 In the same way the weight of vapor vaporizing per unit time would be —

$$wdH = k'aAdx(P' - p) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [4]$$

where k' is the diffusion coefficient in pounds per unit area of exposed surface, P' the vapor pressure of the water, and p the partial pressure of the water vapor in the air. Since for small partial pressures p is nearly proportional to the absolute humidity H, Equation [4] may be rewritten as —

$$WdH = kaAdx(P - H) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [5]$$

where P, the vapor pressure of the water, is expressed in terms of absolute humidity, that is, the absolute humidity of saturated air at the temperature of the water in question.

10 Dividing [5] by [3] gives —

$$\frac{dH}{sdT} = \frac{k(P - H)}{h(T - t)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [6]$$

11 The mechanism by which the heat passes from water to the air may be understood by considering that the heat flows first

from the interior of the water to the surface, and then from the surface through a substantially stationary air film in contact with the surface to the moving air.

12 It has been shown [1] that $h/k = s$ when h refers to the coefficient of heat transfer through the air film only. If the total coefficient of heat transfer from the interior of the water to the moving air be used instead of h, the ratio h/k would not equal s, but would only approximate it to a greater or lesser degree according to whether the heat flow through the water took place easily or with difficulty as compared with the flow through the air film.

13 Unless otherwise noted h and k will henceforth refer to the overall coefficient of heat transfer and vapor diffusion respectively.

14 Therefore, taking $h/k = s$, [6] becomes —

$$\frac{dH}{dT} = \frac{P - H}{T - t} \qquad [7]$$

or —

$$\frac{dH}{P - H} = \frac{dT}{T - t} \qquad [8]$$

Equation [7], stated in words, says that the *differential* increase in humidity of the air is to the *differential* increase in temperature of the air as the humidity difference between the air and the water is to the temperature difference between the air and the water.

15 Equation [7] or [8] when integrated between proper limits would give the change in humidity and temperature of the water in its passage through the cooling tower. However, while the humidity of the air and the vapor pressure of the water are related to each other and the temperatures of the air and water are also related, the relationships are nevertheless of such a nature that exact integration is impossible, and an approximation is obtained by the following assumption.

16 It will be noted that the temperature difference between the water and the air in a counter-current cooling tower does not change greatly between the top and bottom of the tower, nor does the difference in humidities between the water and air. It is therefore a justifiable assumption that the average temperature difference is approximately equal to the arithmetical mean temperature difference, and that the average humidity difference is approximately equal to the arithmetical mean humidity difference. It may therefore

[1] W. K. Lewis, The Evaporation of a Liquid into a Gas, paper No. 1849 of this volume of Trans.

be said that for the whole tower, the total increase in humidity of the air is to the total increase in temperature of the air as the arithmetical mean humidity difference between the air and the water is to the arithmetical mean temperature difference between the air and the water.

17 For the tower shown in Fig. 1 where the subscripts 0 and 1

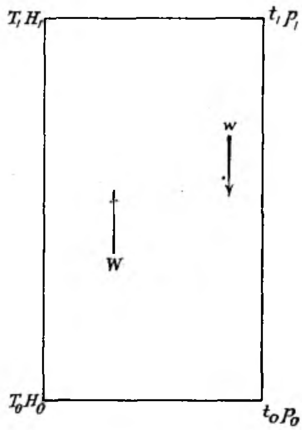

FIG. 1 DIAGRAMMATIC REPRESENTATION OF COUNTER-CURRENT COOLING TOWER

represent conditions at the bottom and top, respectively, there may now be written —

$$\frac{H_1 - H_0}{T_1 - T_0} = \frac{H_1 - P_1 + H_0 - P_0}{T_1 - t_1 + T_0 - t_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad [9]$$

which is the integrated form of Equation [7].

EXPERIMENTAL DATA

18 The author has collected and arranged in Table 1 the results of twenty-three tests made by himself and others on various types of cooling towers.

19 Tests 1 to 5 were on a slat type of forced-draft tower, 6 and 7 were on the same tower operating under natural draft, and Test 23 was on a Wheeler-Balke tower, all described in the Journal of the Ohio Society of Mechanical, Electrical and Steam Engineers, vol. 7.

20 Tests 8 and 9 were on a forced-draft slat-type tower described in the Journal of the American Society of Refrigerating Engineers, vol. 3, 1916–17, p. 32.

TABLE 1 COOLING-TOWER TEST DATA

Test No.	Tower No.	T	T_1	t_1	t_0	Rel. hum.	Gal. per min.	Q	V
1	1	71	90	105	84.7	40	651	110,500	53,900
2	1	72	93	107.8	87.5	60	638	108,000	50,100
3	1	64	96	112	88.5	60	638	124,500	51,400
4	1	69	92	108.5	87	48	643	115,000	52,200
5	1	83	95	109.9	90.5	48	640	103,400	50,600
6	1	43	101	116	98	75	632	94,800	23,500
7	1	60	118	135	115.8	73	630	102,000	15,575
8	2	82	99	115	83	65	617[c]	176,800	72,700
9	2	75	96	109	81	74	668	168,800	72,700
10	3	25	80[a]	87.5	71.3	72	590	78,000	....
11	4	75.8	87.7	95.1	89.1	85	248	12,400	11,100[c]
12	4	77.0	103.9	112.1	99.2	78	230	24,700	9,060[c]
13	4	79.0	112.1	123.3	105.5	76	219	32,400	8,490[c]
14	4	76.1	89.9	94.4	87.9	60	288	15,500	9,700[c]
15	4	76.9	101.7	106.5	95.5	60	278	25,400	8,930[c]
16	4	77.0	112.9	123.7	105.2	57	259	39,800	8,970[c]
17	4	72.1	91.2	94.5	88.0	78	254	13,700	8,090[c]
18	4	73.0	105.1	113.7	99.9	75	246	28,100	9,030[c]
19	4	76.6	115.0	132.1	109.4	68	237	44,700	9,590[c]
20	4	72.9	90.9	92.4	88.4	87	269	8,900	6,160[c]
21	4	72.3	105.4	113.3	100.6	89	257	27,000	8,870[c]
22	4	72.7	115.2	128.2	108.1	88	241	40,200	8,800[c]
23	5	91	106	109	97	59	3,200	319,000	125,000[c]

Test No.	Tower No.	H_0	P_0	H_1	P_1	H	W	Δw	$ka \times$ vol.
1	1	0.006	0.026	0.031	0.050	0.0195	3700	93	4,800
2	1	0.010	0.028	0.034	0.055	0.0195	3410	82	4,200
3	1	0.008	0.029	0.037	0.062	0.023	3460	100	4,300
4	1	0.007	0.028	0.033	0.056	0.022	3560	89	4,000
5	1	0.012	0.031	0.036	0.058	0.0205	3420	82	4,000
6	1	0.004	0.040	0.044	0.071	0.0315	1550	62	2,000
7	1	0.008	0.070	0.075	0.129	0.057	915	61	1,100
8	2	0.015	0.024	0.041	0.068	0.018	5200[c]	135	7,500
9	2	0.014	0.023	0.037	0.057	0.0145	5276[c]	121	8,300
10	3	0.002	0.016	0.022	0.028	0.010	5200[c]	99.5	.....
11	4	0.015	0.030	0.027	0.036	0.012	790[c]	9.5	790
12	4	0.015	0.041	0.045	0.063	0.022	648[c]	19.4	880
13	4	0.016	0.051	0.059	0.090	0.033	606[c]	26.7	810
14	4	0.011	0.029	0.029	0.035	0.012	693[c]	12.5	1,040
15	4	0.011	0.037	0.043	0.052	0.018	638[c]	20.4	1,130
16	4	0.011	0.050	0.061	0.090	0.034	641[c]	32.0	940
17	4	0.013	0.029	0.031	0.036	0.0105	578[c]	10.4	990
18	4	0.013	0.042	0.047	0.066	0.024	644[c]	21.9	910
19	4	0.013	0.058	0.065	0.118	0.049	692[c]	36.0	730
20	4	0.015	0.029	0.030	0.033	0.0085	440[c]	6.6	780
21	4	0.015	0.043	0.048	0.065	0.023	633[c]	20.9	910
22	4	0.015	0.055	0.066	0.104	0.039	628[c]	32.0	820
23	5	0.019	0.039	0.051	0.057	0.013	8250[c]	273.0	21,000

Test No.	Tower No.	Q_{sens}	Δt	$ha \times$ vol.	h/k	ha	ka	u	$100 \times \frac{ka}{u}$
1	1	18,000	14.4	1250	0.26	0.57	2.18	540	0.40
2	1	21,000	15.2	1380	0.33	0.63	1.91	500	0.38
3	1	32,000	20.3	1570	0.36	0.71	1.96	510	0.38
4	1	24,000	17.3	1390	0.35	0.63	1.82	520	0.35
5	1	12,000	10.7	1120	0.28	0.51	1.82	510	0.36
6	1	26,000	35	740	0.37	0.34	0.91	240	0.38
7	1	15,000	36.4	410	0.37	0.19	0.50	160	0.31
8	2	23,000	24.5	1470	0.20	0.46	2.36	320	0.77
9	2	29,000	25	1720	0.21	0.54	2.61	320	0.82
10	3	......	....	....	....	....	....	...	....
11	4	2,400	10.4	230	0.29	0.17	0.57	174	0.33
12	4	4,300	15.2	283	0.32	0.20	0.64	142	0.45
13	4	4,400	18.9	233	0.29	0.17	0.59	133	0.44
14	4	2,400	8.2	292	0.28	0.21	0.75	151	0.50
15	4	4,000	11.7	342	0.30	0.25	0.82	140	0.58
16	4	6,200	19.5	318	0.34	0.23	0.68	140	0.49
17	4	2,800	9.6	292	0.30	0.21	0.72	126	0.57
18	4	5,100	18.9	270	0.30	0.21	0.66	141	0.47
19	4	6,900	25.0	276	0.38	0.20	0.53	150	0.35
20	4	2,000	8.4	238	0.31	0.17	0.57	96	0.59
21	4	5,000	18.1	276	0.30	0.20	0.66	139	0.47
22	4	6,600	24.2	273	0.33	0.20	0.59	125	0.47
23	5	31,000	4.5	7,000	0.33.	....	....	...	....

[a] Assumed values. [c] Calculated values.

21 Test 10 was on a natural-draft tower described in the Transactions of The American Society of Mechanical Engineers, vol. 31, 1909, p. 75.

22 Tests 11 to 22 were on an experimental Badger spray-type tower erected at the Massachusetts Institute of Technology, and the results have not been previously published.

23 The method of calculating the data may be explained by reference to Test 1.

T_0 = temperature of inlet air, deg. fahr.

T_1 = temperature of outlet air, deg. fahr.

t_1 = temperature of inlet water, deg. fahr.

t_0 = temperature of outlet water, deg. fahr.

Rel. Hum. = relative humidity in per cent

Gal. Min. = gallons of water circulated per minute

Q = heat given up by water in B.t.u. per minute

$= 651 \times 8.3 \times (105 - 84.7) = 110{,}500$

V = cubic feet of air per minute

H_0 = absolute humidity of inlet air in pounds of water per pound of dry air, as read from humidity chart [1]

P_0 = equivalent absolute humidity of outlet water, i.e., the absolute humidity of air saturated at the water temperature

H_1 = absolute humidity of outlet air, assuming saturation

P_1 = equivalent absolute humidity of inlet water

H = mean humidity difference between the air and water

$$= \frac{0.026 - 0.006 + 0.050 - 0.031}{2} = 0.0195$$

W = pounds of dry air per minute, as calculated from the volume of air V divided by the humid volume (the cubic feet occupied by one pound of dry air plus the water vapor which it contains, as read from the humidity chart)

w = pounds of water evaporated per minute

$w = W(H_1 - H_0) = 3700(0.031 - 0.006) = 93$

$ka \times$ vol. = pounds of water evaporated in the whole tower per minute per one pound mean humidity difference

$$= \frac{93}{0.0195} = 4800$$

[1] Wm. S. Grosvenor, Trans. Am. Inst. Chem. Engrs., 1908.

$$
\begin{aligned}
Q_{sen} &= \text{sensible heat picked up by the air per minute} \\
&= Ws(T_1 - T_0), \text{ where } s \text{ is the humid heat as read from the humidity chart[1] } (s = 0.25) \\
&= 3700 \times 0.25(90 - 71) = 18{,}000 \text{ B.t.u.} \\
t &= \text{mean temperature difference between air and water} \\
&= (105 - 90 + 84.7 - 71) / 2 = 14.4 \\
ha \times \text{vol.} &= \text{B.t.u. picked up as sensible heat by air in the whole tower per degree temperature difference} \\
&= \frac{18{,}000}{14.4} = 1250 \\
\frac{h}{k} &= \frac{ha \times \text{vol.}}{ka \times \text{vol.}} = \frac{1250}{4800} = 0.26 \\
ha &= \frac{ha \times \text{vol.}}{\text{vol.}}, \text{ where ``vol.'' is the volume of the tower.}
\end{aligned}
$$

24 The dimensions of the tower in Test 1 were not given in the article, but other information led to the assumption that the tower was 10 × 10 × 22 ft., a volume of 2200 cu. ft., whence —

$$
\begin{aligned}
ha &= \frac{1250}{2200} = 0.57 \text{ and } ka = \frac{4800}{2200} = 2.18 \\
u &= \text{linear velocity of air through the total cross-section of the tower} \\
&= \frac{53{,}900}{10 \times 10} = 540 \text{ ft. per. min.} \\
\frac{ka}{u} &= \frac{2.18}{540} = 0.0040
\end{aligned}
$$

25 The most interesting tests are those numbered 1 to 7, which have been quoted several times in articles by different investigators. In these tests the first five were under forced draft, while the last two were under natural draft, with much reduced air velocity. A plot of *ka* versus air velocity for these seven tests gives, as shown in Fig. 2, a straight line passing through the origin.

26 It has been noticed by others [2], that the rate of cooling in towers is approximately proportional to the air velocity. Investigations at the Institute being carried on at the present time indicate that *ka* for humidifying apparatus in general is probably proportional to a power function of the air velocity something less than one, but

[1] Wm. S. Grosvenor, Trans. Am. Inst. Chem. Engrs., 1908.

[2] Jour., Am. Soc. Refrig. Engrs., vol. 3, 1916–17, p. 32.

for the present, at least, linear proportionality is a fairly close approximation. While k is the actual coefficient of diffusion per square foot of surface, since in many types of towers the active surface cannot be accurately measured the coefficient ka is used instead, where a represents the actual surface in square feet per cubic foot of tower. In most towers a is uniform throughout the volume of the tower.

27 It is therefore justifiable to call the value ka/u the "tower constant" and to use the value of this constant as a means for comparing the operation of towers of various sizes and types. Thus for tower No. 1 the average value of ka/u is 0.0037 and the average deviation from this value is 0.00021, or less than 6 per cent, while the maximum deviation is 0.0006, or 16 per cent. The important

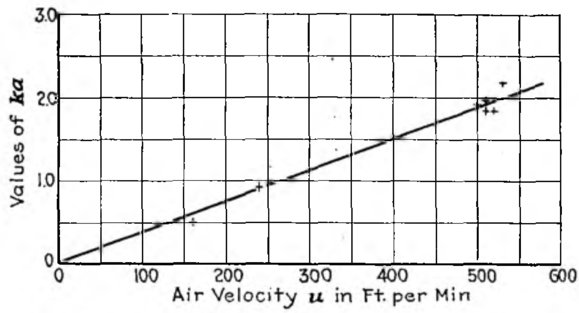

FIG. 2 RELATION BETWEEN ka AND AIR VELOCITY

thing indicated by these tests is that ka/u was unaffected whether the tower was operated with forced or with natural draft.

28 Tower No. 4 has an average value of 0.00475 for ka/u, with an average deviation of 10 per cent and a maximum deviation of 31 per cent.

29 Tower No. 2 has an average constant of 0.0080, which is high compared with the other towers tested. The drop in pressure through this tower was not published. It would be interesting to compare its drop in pressure with that in other towers which show smaller tower constants. It is of course obvious that a high tower constant can be obtained by obstructing the flow of air through the tower by cutting down the mean free area, but this again is at the expense of friction and back pressure. Wherever possible the drop in pressure through the tower should be measured and published.

30 The average value of the humid heat of air in a cooling

tower is about 0.25. It will be noted that the column h/k in Table 1, which should be approximately equal[1] to s, has values of the same order of magnitude in general, but which are somewhat higher. The reason for this as predicted from the statements made in Par. 12 is shown by the fact that, while the surface of the water is at a lower temperature than the interior, making the true h for the air film greater, the value of k was calculated for the average water temperature instead of using the surface temperature. The latter is lower and would give a higher value for k. But since the vapor pressure of water rises more rapidly than the temperature, the ratio of h to k calculated on the average water temperature would be greater than that calculated on the lower surface temperature, which was the case in all but two cases in Table 1. The experimental data are therefore offered as proof of the validity of Equation [**7**].

31 Information regarding the size of tower in Test 23 was not available. It would be interesting to compare the values of ha and ka for this large tower with those of the smaller ones in the previous tests.

32 The results of the calculations of the test runs are felt to confirm in a remarkable manner the conceptions regarding humidification in general as developed by W. K. Lewis, furnishing as they do ample experimental proof of their validity.

APPLICATION OF THE EQUATIONS DEVELOPED TO TOWER DESIGN

33 The value of the foregoing equations with respect to cooling-tower design may best be shown by means of an example.

34 In any cooling tower the law of conservation of energy must apply. This law is represented by the heat balance of Equation [**1**]. Furthermore, in any cooling tower there is a necessary relationship between the amount of heat transferred by conduction and that eliminated by evaporation which is represented by Equation [**9**]. Finally, the capacity of any cooling tower is determined by the rate of transfer of heat and the rate of diffusion of vapor in it. These rates are dependent upon the design of the tower, and can only be determined on the basis of experimental data on the performance of towers of the type in question. This capacity factor is covered by Equation [**3**] or Equation [**5**] in the differential form, but in actual

[1] W. K. Lewis, The Evaporation of a Liquid into a Gas, paper No. 1849 of this volume of Trans.

design it is more satisfactory to use an integrated form of Equation [5] obtained by the use of arithmetical mean humidity differences and represented by Equation [10]. The combination of these three equations represents the conditions that must obtain in any tower and therefore serve as a proper basis for tower design.

35 The coefficient ka/u expresses the volumetric capacity of any particular type of tower. Experimental determination of this constant is necessary before design can be accomplished.

36 It is necessary for the engineer to select the type of tower most suitable for his purpose, and, from previous tests on towers of similar type, obtain values of ka which can be anticipated. In general certain specifications must be met. These may be:

Weight of water to be cooled, w
Temperature of water to be cooled, t_1
Temperature to which water must be cooled, t_0
Average (or worst) outside air temperature, T_0
Average (or worst) outside air humidity, H_0

for which the following values may be taken:

w = 3000 gal. per min.
t_1 = 110 deg. fahr.
t_0 = 80 deg. fahr.
T_0 = 80 deg. fahr.
H_0 = 0.0130 (60 per cent relative humidity)

those for T_0 and H_0 being the worst atmospheric conditions under which a tower must cool the water at 80 deg. fahr.

37 It will be noted that the condition of reducing the temperature of the water to that of the entering air is exceptionally severe and will call for a tower considerably larger than usual, since towers rarely have to meet such specifications.

38 There are in this case three unknown conditions, the temperature and humidity of the outgoing air, and either the volume of the tower or the ratio of the amount of air to the amount of water. The air-water ratio is usually determined by the type of tower selected and is therefore known, leaving the volume of the tower as the third unknown.

39 In order to calculate these unknown quantities three independent equations are needed. These may be taken as Equation [2],

$$w(t_1 - t_0) = Ws(T_1 - T_0) + Wr(H_1 - H_0)$$

Equation [**9**],

$$\frac{H_1 - H_0}{T_1 - T_0} = \frac{H_1 - P_1 + H_0 - P_0}{T_1 - t_1 + T_0 - t_0}$$

and an integrated form of Equation [**4**], obtained by employing the same assumption as that used in integrating Equation [**7**],

$$W(H_1 - H_0) = kaAx\,\frac{P_1 + P_0 - H_1 - H_0}{2} \quad . \; . \; . \; . \quad [\mathbf{10}]$$

Equations [**2**], [**9**], and [**10**] may be solved simultaneously for H_1, T_1, and Ax (which latter equals the volume of the tower). The author has seen fit to solve for W instead of Ax, but after H_1 and T_1 have been found it is easy to convert the solution for W into that for Ax. The final values are as follows:

$$T_1 = -T_0 + t_1 + t_0 - \frac{\dfrac{2w(t_1 - t_0)}{kaAx}}{s + r\dfrac{2H_0 - P_1 - P_0}{2T_0 - t_1 - t_0}} \quad . \; . \; . \; . \; . \quad [\mathbf{11}]$$

$$H_1 = -H_0 + P_1 + P_0 - \frac{\dfrac{2w(t_1 - t_0)}{kaAx}}{s\dfrac{2T_0 - t_1 - t_0}{2H_0 - P_1 - P_0} + r} \quad . \; . \; . \; . \quad [\mathbf{12}]$$

$$W = -\frac{\dfrac{w(t_1 - t_0)}{s(2T_0 - t_1 - t_0) + r(2H_0 - P_1 - P_0)}}{1 + \dfrac{2w(t_1 - t_0)}{kaAxs(2T_0 - t_1 - t_0) + r(2H_0 - P_1 - P_0)}} \quad . \quad [\mathbf{13}]$$

These equations become much simplified when the known numerical values are substitutes for the literal quantities.

40 In the particular problem in question, suppose that a forced-draft slat-type tower such as was used in Tests 1 to 7, whose tower constant ka/u is 0.0037, be selected. Towers of this type are found to be economical when handling 6.5 gal. of water per min. per sq. ft. of ground area with an air velocity of about 500 ft. per min.

41 The area of the proposed tower will therefore be 3000/6.5 = 460 sq. ft., and the volume of air will be 500 × 460 = 230,000 cu. ft. per min. The humid volume of one pound of dry air as read from the humidity chart is 13.8 cu. ft. Therefore —

$$W = \frac{230{,}000}{13.8} = 16{,}700 \text{ lb. per min.}$$

also —

$w = 3000 \times 8.3 = 25{,}000$ $\quad r = 1050$ (approx.)
$t_1 = 110$ $\quad H_0 = 0.013$
$t_0 = 80$ $\quad P_1 = 0.0585$
$s = 0.24$ $\quad P_0 = 0.022$
$T_0 = 80$ $\quad ka = 0.0037 \times 500 = 1.85$

Substituting these values in Equation [**13**] and solving for Ax gives 41,500 cu. ft. for the volume of the tower, and since the ground area was determined to be 460 sq. ft., the tower height would be 41,500/460 = 90 ft. The severe conditions imposed account for the great height required.

42 If the tower be built as calculated, the performance under any other atmospheric conditions may be readily calculated by substitution in the proper equation.

43 The author realizes that the foregoing methods of calculating cooling-tower performance do not form the complete solution of the problem, and that the ultimate design of the best tower will depend upon the striking of an economic balance between the size of the tower and the cost of moving the water and the air. He feels, however, that this method of calculation is distinctly in advance of anything which has thus far appeared in print, that it furnishes a convenient and accurate tool for the designing engineer, and that it is the necessary basis for the economic balance referred to.

44 Finally, the author wishes to urge the inclusion of more complete and more accurate data in published accounts of tower performances. Of all the published tests studied, only those tabulated had sufficient data to enable them to be analyzed, and even then, in most cases, assumptions were necessary. Inaccurate data are often common. Test No. 10 is an example of this, the heat of vaporization alone being considerably greater than the total cooling of the water, which inaccuracy renders the test useless for purposes of analysis. A more complete knowledge of the effect of varying conditions on ha and ka can only come from studies of large numbers of accurate tests, and it is upon such increased knowledge that advance in tower design depends.

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DISCUSSION

B. H. Coffey. The members of the American Society of Refrigerating Engineers present will remember that Mr. George Horn and the writer have devoted much time and study to this subject. We are therefore gratified to see the theory of cooling towers becoming of interest to the scientific men of college faculties as instanced by the present paper. If the unequalled experimental facilities and mathematical ability of our great engineering schools become engaged upon this subject, we believe it can be shortly put upon a practical basis for the general profession.

All cooling towers are mere structural assemblages of cooling surface, the cost of which is by far the largest item in the installation. For this reason the designer and purchaser are peculiarly interested in the area of surface required to do the specified cooling. The discussion will therefore be confined to the points bearing on this part of the subject.

Equations [**3**] and [**4**] show the dual heat currents that always exist between air and water when not in thermal equilibrium, both expressions containing a cooling surface term and transmission coefficient. In Equation [**3**] the potential or driving force is temperature difference and in Equation [**4**] pressure difference. For future reference we wish here to point out that by simple transposition in each equation the surface increment equals the heat increment multiplied by the reciprocal of the potential and consequently for zero potential the surface increment is infinite.

By ingenious use of the connecting constant s, Equation [**8**] is established showing the relation between the latent and sensible heat currents. The author fails to integrate this equation and resorts to an approximation with which we take issue.

The approximation (Par. 16) is based upon the assumption that the mean of the extreme differences is the true mean and the assumption is based upon the stated fact that in counter-current cooling towers the pressure and temperature differences do not change greatly between the top and bottom of the tower. No evidence is offered to support this statement which we dispute. Referring to Par. 20, Table 1, Tests 1 to 7, the temperature differences at the bottom of the tower as a percentage of those at the top range from 91 to 366 per cent and on the same basis the pressure differences vary from 83 to 148 per cent. We think these figures are sufficient to cast grave doubts upon the author's statement and basis of his approximation and that probably the mean potentials he obtains are not the true means.

If the mean potentials are incorrect the mean transmission coefficients h and k which are derived from them (Par. 23) are incorrect and the ratio h/k will be incorrect. Referring to Table 1, col. h/k, we find this ratio varies from 0.20 to 0.38 when it should be approximately constant at 0.25 (Par. 30). The explanation of these wide discrepancies given in this paragraph are to us not convincing.

The extraordinary height of tower 90 ft. (Par. 41) to meet the not particularly severe conditions of Par. 36 as obtained from the simultaneous Equation [**13**], Par. 39, is, we believe, due to incorrect mean potentials.

As a further test of Equation [**13**] we used for final water temperature 69.7 deg. the wet-bulb temperature of the entering air. Under these conditions the total potential becomes zero and the cooling surface infinity as pointed out above. The calculation resulted in a tower 230 ft. high instead of infinity which we again attribute to incorrect mean potentials.

In our opinion this line of attack upon the cooling tower problem while promising has failed to produce working formulas that express the cooling process. We suggest as a basis using total potential equations. The relation $h/k = s$ gives a means of converting temperature potential into equivalent pressure potential or the reverse. The total potential can be thus expressed either in temperature or pressure alone and the problem much simplified.

We note with regret the absence of any reference to the wet-bulb temperature in this paper, a physical quantity universally regarded of the greatest importance in this subject.

W. M. Grosvenor.[1] It is a real satisfaction to find after fourteen years that a piece of one's own work stands the test of time and is still of use to other engineers, particularly in such a very admirable consideration of cooling towers as the author has given us. The article on Calculations for Dryer Design to which he refers was published in the *Proceedings* of the American Institute of Chemical Engineers and the *Heating and Ventilating Magazine* for 1908 when the best available data on humidity were those of the U. S. Weather Bureau and on these figures the calculations were based and the resulting curves plotted. The conception was

[1] Cons. Chemist and Factory Engr., 50 E. 41 Street, New York, N. Y.

there introduced of representing what might be called adiabatic evaporative cooling by lines intersecting the curves of relative humidity on a chart having temperatures as one ordinate and weight of moisture per lb. of air, volume per lb., B.t.u. per lb. of air when damp (humid heat), etc., on the other ordinate. This has proven a very useful and easy way of calculation. Some three years later at the Annual Meeting of The American Society of Mechanical Engineers for 1911 in a paper entitled Rational Psychrometric Formulae, Willis H. Carrier used this method of presenting a newly calculated set of curves based, I believe, on more accurate data than those of the Weather Bureau. It seems to the writer that he did a very excellent and valuable piece of research work that was very thoroughly discussed at that meeting but has received too little attention since. This valuable paper contains no reference to any previous publication of humidity charts with adiabatic cooling lines, etc., but the writer wants to call Mr. Robinson's attention to Mr. Carrier's paper and to ask for it the careful consideration it apparently deserves.

Now that we have in the author's work the foundation laid for a more logical and clear understanding of the data needed for perfecting the design of cooling towers, it becomes the obvious duty of engineers having cooling towers under operation to determine as well as they can the conditions of operation and communicate the information to him. He is then in position to suggest changes in operating conditions if not in design and, on the basis of the results before and after, can revise and perfect what should be a very valuable solution of a problem that is becoming increasingly important with municipal growth.

The Author is glad of an opportunity to clear up an obscure point in his paper, as indicated by Mr. Coffey's discussion.

It is correct, as pointed out by Mr. Coffey, that the differences between the temperatures of the air and the water at the top and at the bottom of the tower in Tests 1 to 7 vary by as much as four fold in the extreme case. The statement of the author that the temperature differences remained substantially constant therefore seems far from the fact. However, the author had in mind when he made the statement, the fact that the integral of the Equation [8] requires the true mean temperatures and humidity differences. Not knowing the true means, and realizing that the differences varied not over four fold, it seemed perfectly proper to assume the arithmetical mean temperature differences, since they would probably be not very far from the true mean. He based this statement on the fact that the log mean temperature difference would give a result only differing 16 per cent from the arithmetical mean difference when the difference is four fold, while the geometric mean difference would differ by less than 20 per cent, when the difference in the temperatures themselves varied by over 366 per cent. This does not mean that either of these two means is correct in this case, but it does show that in estimating mean values where the values themselves do not vary by more than four fold, the arithmetical mean is likely to be well within the experimental error of the tests.

The author therefore believes that within the limits of commercial requirements, his solution of the equation is practicable and satisfactory, and he does not believe that the error introduced by this approximation is the chief cause of the variation in the values of h/k obtained. The use of the arithmetical mean has the obvious effect, as pointed out by Mr. Coffey, of giving a finite answer when an infinite one is required by the theory, but since an infinite tower is beyond the range of actual practice, he still feels that the equations developed go at least one step further towards the solution of the cooling tower problem than anything that has heretofore appeared in print.