Elements of Milling, Part 2

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The results are given of a series of experiments in which a single-point tool corresponding to a single tooth of a milling cutter was used in removing metal in a dynamometer of a pendulum type. The values of energy as a function of variable feed, depth, width, and front- and side-rake angles of the cutter were determined when cutting three steels and a brass, in both groove and land cutting. It is found that all data obtained follow the general equation for energy in which $E = Cwf^xd^y$. It is shown that invariably groove cutting requires greater energy than land cutting. It also is shown that greater values of front rake on the cutter reduce the energy required to remove a chip, while the value of side rake for a given value of front rake exerts no appreciable influence on energy values. The results of some 24,000 commercial tests made by students at the Ohio State University in cooperation with the Cincinnati Milling Machine Company are presented and analyzed. The results are shown to agree substantially with those obtained with the single-point tool and that the foregoing energy equation holds for all types of milling, such as slabbing, groove cutting, half-side milling, and facing.

THE object of this paper is to extend the tests reported on in the previous paper, "Elements of Milling,"[3] presented before the Society in December, 1931.

In that paper, the variables studied, as they affect energy, were feed per tooth, depth of cut, width of cut, cutting up versus cutting down, and cutting fluids. Those tests were run on a number of ferrous and non-ferrous metals, including Bakelite and wood fiber. All cuts were made to form a groove in the work. The tool shape was constant, being end-cutting and having a 15-deg rake with no side rake.

This paper contains the results of experiments with cutters having variable back-rake angles, or hook, and side-rake angles, or helix. Tests with each cutter were made when cutting in a groove and when cutting the top from a land which had been prepared previously. Several types of metals were included, all cutting being done with one cutting fluid—namely, an emulsion consisting of 1 part soluble oil to 50 parts water, No. 3.

Power data also are presented from a series of experiments on steel and cast iron, in which various types of commercial milling cutters were used as obtained by cooperative research by students at Ohio State University for the Cincinnati Milling Machine Company. The results have been recomputed to a basis similar to that used by the authors, for the purpose of direct comparison.

Cutters and Equipment

The machine used in these tests was the same as used and described previously in "Elements of Milling." This machine is of the pendulum type, such as an impact testing machine, and uses a single-tooth cutter. The cutters in all cases were adjusted to have a radius of 1.75 in., and all had end- and side-clearance angles of 4 deg. Nine series of cutters were used, divided into two groups. The first group, consisting of five cutters with 0-deg helix angles and with rake angles of 0, 10, 15, 20, and 25 deg, were designed to study the influence of variable rake. The second group had rake angles of 15 deg and helix angles of 10, 20, 30, and 40 deg, which, together with a 0-deg helix cutter having 15-deg rake angle, were used to study the influence of variable helix angles.

High-speed-steel blades $^3/_8$ in. wide by $^3/_4$ in. deep were used. They first were ground all over on a surface grinder, after which the various cutting angles were accurately ground on the same machine. All cutters were 0.300 in. wide. As this width decreased slightly from subsequent grindings, the data were corrected for original width on the assumption that the energy varies directly as the width. This assumption had been proved previously.

Materials and Cutting Fluids Used

The first material tested was a free-cutting leaded brass and was selected because it was believed that more accurate data could be obtained using a metal which would not cause rapid dulling of the cutter. Three other steels then were tested: S.A.E. 1020, S.A.E. 1112, and S.A.E. 3250. These metals have been described previously and are listed in Table 1, together with their chemical compositions, heat treatments, and hardness values.

The cutting fluid used consisted of 1 part soluble oil in 50 parts water, and is listed and described as cutting fluid No. 3 in the previous paper.

Experimental Method and Scope of Test

As in the previous paper, all cuts were made with work locked in position as the chip was removed. Tests in all cases were made both cutting up (that is, with the work fed against the cutter) and cutting down (that is, with the work fed with the cutter). All of the cuts were taken both in a groove and on a land to determine the relation between the two types of cuts.

Each test value indicated is the average of from 10 to 30 cuts, and the data presented represent a total of some 6000 or 7000 cuts. Great care was taken to keep the cutter in the sharp condition. By sharp is meant the edge produced by the surface grinder with the final slight burr removed by a hand hone.

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[3] Trans. A.S.M.E., vol. 54, 1932, paper No. RP-54-4.

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TABLE 1 MATERIALS USED IN THE MILLING TESTS

Bar number	Material	Chemical analysis	Heat treatment	Hardness Brinell	Hardness Rockwell B	Hardness Scleroscope
BF	Leaded screw-stock brass	62% Cu, 34% Zn, 3% Pb	Extruded and cold-drawn	70	66.0	..
EF	S.A.E. 1020 steel	0.22 C, 0.52 Mn, 0.011 P, 0.026 Si	Annealed	131	65.0	..
MF	S.A.E. 1112 steel	0.09–0.13 C, 0.7–0.9 Mn, 0.08–0.40 P, 0.085–0.12 S	Cold-drawn	217	89.5	..
A-7	S.A.E. 3250 steel	Commercial	Normalize 1650 F, anneal 1475 F	207	92.5	30

TABLE 2

(Energy values in foot-pounds per chip when milling a free-cutting brass, bar *BF*, up and down, in a groove, and on a land for various combinations of feed and depth of cut with several different shaped cutters. The width of each cutter was 0.300 in., and an emulsion consisting of 1 part soluble oil to 50 parts water was used as a cutting fluid)

Tool angles	Feed in inches	Depth in inches	Cutting in groove Up	Cutting in groove Down	Cutting on land Up	Cutting on land Down
0-deg helix, 0-deg rake	0.020	0.010	0.74	0.68	0.74	0.70
		0.025	1.76	1.68	1.66	1.57
		0.050	3.48	3.23	3.10	2.88
		0.100	6.80	6.29	5.81	5.62
	0.010	0.100	4.08	3.73	3.55	3.23
	0.030		9.69	9.11	8.18	7.83
	0.040		11.99	11.38	9.88	9.57
0-deg helix, 10-deg rake	0.020	0.010	0.66	0.59	0.57	0.50
		0.025	1.62	1.39	1.40	1.26
		0.050	3.14	2.73	2.80	2.50
		0.100	6.12	5.54	5.52	5.02
	0.010	0.100	3.45	3.12	3.14	2.84
	0.030		8.87	3.97	7.60	7.15
	0.040		11.53	10.29	9.88	9.14
0-deg helix, 20-deg rake	0.020	0.010	0.68	0.62	0.57	0.52
		0.025	1.55	1.43	1.32	1.19
		0.050	2.88	2.62	2.50	2.25
		0.100	5.32	4.97	4.73	4.27
	0.010	0.100	3.03	2.75	2.77	2.47
	0.030		7.50	6.95	6.59	5.97
	0.040		9.81	8.90	8.66	7.66
0-deg helix, 25-deg rake	0.020	0.010	0.66	0.60	0.65	0.58
		0.025	1.47	1.34	1.42	1.31
		0.050	2.78	2.47	2.52	2.24
		0.100	5.11	4.76	4.64	4.22
	0.010	0.100	2.95	2.65	2.74	2.46
	0.030		7.22	6.72	6.43	5.78
	0.040		9.31	8.61	8.26	7.36
0-deg helix, 15-deg rake	0.020	0.010	0.64	0.56	0.60	0.53
		0.025	1.56	1.33	1.38	1.24
		0.050	3.03	2.64	2.63	2.33
		0.100	5.86	5.04	4.94	4.50
	0.010	0.100	3.20	2.76	2.83	2.56
	0.030		8.27	7.18	7.00	6.30
	0.040		10.41	9.15	8.85	8.10
10-deg helix, 15-deg rake	0.020	0.010	0.80	0.56	0.67	0.61
		0.025	1.69	1.50	1.55	1.46
		0.050	3.20	2.85	3.14	2.87
		0.100	6.26	5.53	6.03	5.57
	0.010	0.100	3.55	3.08	3.48	3.25
	0.030		8.88	7.82	8.58	7.76
	0.040		11.40	10.00	11.13	9.94
20-deg helix, 15-deg rake	0.020	0.010	0.74	0.65	0.73	0.65
		0.025	1.72	1.54	1.66	1.48
		0.050	3.33	2.92	3.24	2.81
		0.100	6.20	5.51	5.93	5.35
	0.010	0.100	3.44	3.06	3.41	3.07
	0.030		8.85	7.85	8.30	7.65
	0.040		11.10	10.01	10.90	9.85
30-deg helix, 15-deg rake	0.020	0.010	0.72	0.61	0.69	0.62
		0.025	1.73	1.51	1.56	1.49
		0.050	3.45	2.96	3.15	2.74
		0.100	6.85	5.90	5.70	5.18
	0.010	0.100	3.90	3.41	3.23	2.95
	0.030		9.56	8.55	8.05	7.45
	0.040		12.40	11.10	10.35	9.50
40-deg helix, 15-deg rake	0.020	0.010	0.60	0.55	0.58	0.55
		0.025	1.57	1.43	1.37	1.28
		0.050	3.41	3.07	2.66	2.53
		0.100	6.94	6.37	5.10	4.86
	0.010	0.100	3.70	3.42	2.88	2.60
	0.030		9.84	9.10	7.33	7.00
	0.040		12.73	11.68	9.66	9.02

The values presented, therefore, are somewhat lower than could be expected from the average cutter condition. The authors have found that consistent results can be obtained in no other way, as the degree of cutter wear is almost impossible to measure and has such a decided effect upon the energy values when milling.

Results of Tests

The experimental data for the four metals tested are presented separately below.

Tests on Free-Cutting Brass (*Bar BF*). The experimental data for the leaded free-cutting brass are listed in Table 2. Similar tables have been made up for each of the other materials, but are not shown. As in the first paper, these values are plotted on log-log paper to determine the milling-energy formulas for each tool and method of cut.

Lines have been drawn through the energy data plotted on log-log paper in Fig. 1. One set of lines is for constant feed by variable depth, while the other set is for constant depth but variable feed. Lines also are shown for milling up and milling down on a land in each case. From these lines, an equation for energy in foot-pounds per chip, as a function of the depth of cut and feed, is determined. The tangent of the angle between the line and the horizontal represents the exponent of the variable involved.

Formulas for a variety of shapes of tools and methods of cutting are listed for brass in Table 3, together with the values of

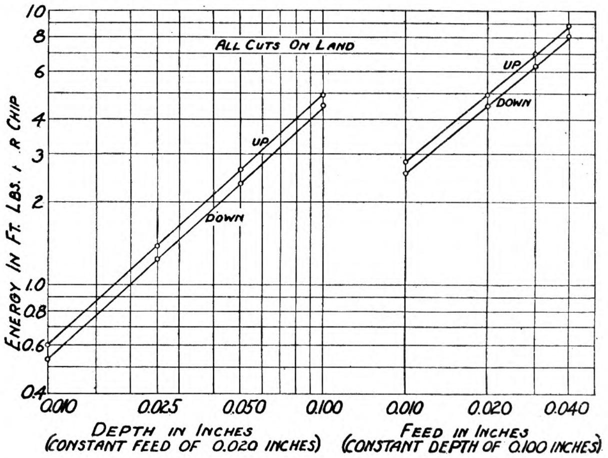

Fig. 1 Energy Required to Mill a Free-Cutting Brass Using a Soluble Oil Consisting of 1 Part Oil to 50 Parts Water

(One series of cuts was made with a constant feed of 0.020 in. as the depth was varied, while the second was made with a constant depth of 0.100 in., while the feed was varied. The cutter was of an end-cutting type having a 15-deg rake angle and no helix. The following formula was obtained: $E = Cwf^{0.83}d^{0.94}$ in which $C_{(\text{up milling})} = 3750$, and $C_{(\text{down milling})} = 3392$.)

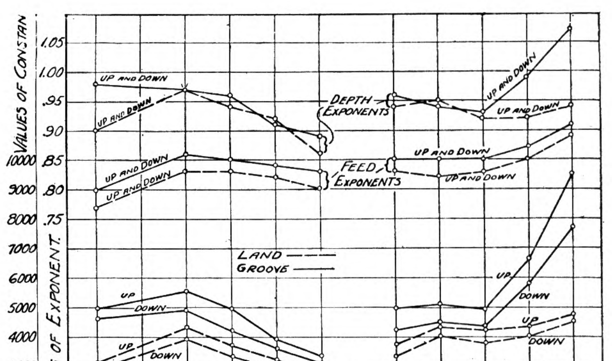

Fig. 2 Effect of Various Rake and Helix Angles of the Cutter on the Exponents of Feed and Depth and the Constant of the Milling-Energy Equation When Milling Free-Cutting Brass, Bar *BF*

(Data taken from Table 3.)

TABLE 3 RESULTS OF TESTS WHEN MILLING FREE-CUTTING BRASS

(An emulsion of 1 part soluble oil to 50 parts water, cutting fluid No. 3, was used. The width of the cutter was 0.300 in.)

Tool angles, deg Helix	Tool angles, deg Rake	Method of cutting	Formulas, energy E, in foot-pounds per chip Groove	Formulas, energy E, in foot-pounds per chip Land	Values of C Groove	Values of C Land	Horsepower per cu in. per min formulas Groove	Horsepower per cu in. per min formulas Land	Horsepower per cu[a] in. per min Groove	Horsepower per cu[a] in. per min Land
0	0	Up	$Cwf^{0.8}d^{0.98}$	$Cwf^{0.77}d^{0.9}$	4970	3124	$\frac{C}{33000\,f^{0.20}d^{0.02}}$	$\frac{C}{33000\,f^{0.23}d^{0.10}}$	0.3960	0.3442
		Down			4670	2984			0.3750	0.3285
0	10	Up	$Cwf^{0.36}d^{0.97}$	$Cwf^{0.83}d^{0.97}$	5575	4360	$\frac{C}{33000\,f^{0.14}d^{0.03}}$	$\frac{C}{33000\,d^{0.17}d^{0.03}}$	0.3452	0.3100
		Down			4905	3942			0.3040	0.2800
0	15	Up	$Cwf^{0.85}d^{0.96}$	$Cwf^{0.83}d^{0.94}$	4975	3750	$\frac{C}{33000\,f^{0.15}d^{0.04}}$	$\frac{C}{33000\,f^{0.17}d^{0.06}}$	0.3290	0.2855
		Down			4260	3392			0.2820	0.2580
0	20	Up	$Cwf^{0.84}d^{0.91}$	$Cwf^{0.82}d^{0.92}$	3905	3240	$\frac{C}{33000\,f^{0.16}d^{0.09}}$	$\frac{C}{33000\,f^{0.18}d^{0.08}}$	0.3040	0.2700
		Down			3630	2925			0.2825	0.2440
0	25	Up	$Cwf^{0.83}d^{0.89}$	$Cwf^{0.80}d^{0.86}$	3380	2580	$\frac{C}{33000\,f^{0.17}d^{0.11}}$	$\frac{C}{33000\,f^{0.20}d^{0.14}}$	0.2888	0.2713
		Down			3115	2365			0.2660	0.2484
10	15	Up	$Cwf^{0.85}d^{0.94}$	$Cwf^{0.82}d^{0.95}$	5120	4340	$\frac{C}{33000\,f^{0.15}d^{0.16}}$	$\frac{C}{33000\,f^{0.18}d^{0.05}}$	0.3465	0.3380
		Down			4540	4090			0.3078	0.3182
20	15	Up	$Cwf^{0.85}d^{0.93}$	$Cwf^{0.83}d^{0.92}$	4915	4240	$\frac{C}{33000\,f^{0.15}d^{0.07}}$	$\frac{C}{33000\,f^{0.17}d^{0.08}}$	0.3490	0.3380
		Down			4385	3800			0.3110	0.3030
30	15	Up	$Cwf^{0.87}d^{0.99}$	$Cwf^{0.85}d^{0.92}$	6690	4355	$\frac{C}{33000\,f^{0.13}d^{0.01}}$	$\frac{C}{33000\,f^{0.15}d^{0.08}}$	0.3775	0.3162
		Down			5800	4060			0.3275	0.2950
40	15	Up			9540	4785	$\frac{Cd^{0.07}}{33000\,f^{0.09}}$	$\frac{C}{33000\,f^{0.11}d^{0.06}}$	0.3722	0.2760
		Down	$Cwf^{0.91}d^{1.07}$	$Cwf^{0.89}d^{0.94}$	8720	4515			0.3400	0.2609

[a] 0.010 in. feed, 0.100 in. depth, and 0.300 in. width.

the constants, the formulas for horsepower per cubic inch of metal removed per minute, and the computed horsepower per cubic inch per minute value for a specific cut 0.100 in. deep, 0.010 in. feed per tooth, and 0.300 in. wide. The last values are included in the table in order to furnish a basis of comparison between the different conditions. It is impossible directly to compare constants or exponents from the equations with energy and horsepower values computed therefrom, because of the several variables involved in the formulas.

It will be noticed from Table 3 that no difference in the formula was found for up and down milling with a single cutter, except in the value of the constant. Changing the tool angles, however, does change the exponents of the feed and depth of cut.

Better to indicate the effect of different angles on the exponents and constants, Fig. 2 has been prepared. In the upper left part of Fig. 2 are shown the depth and feed exponents for both up and down milling for variable rake angles with a constant helix angle of 0 deg. The effect on these exponents when the helix angle is varied is shown on the right, the front rake angle being constant at 15 deg. In the lower portion of Fig. 2 are shown plotted the constants obtained to show the effect of varying the angles.

A study of these curves indicates several interesting characteristics. Except for a few values, all exponents and constants are lower for land cutting than for groove cutting. This, of course, would be expected of the constants, inasmuch as in groove cutting it is necessary to free the chip along the sides of the groove as well as on the bottom. Also, in general the difference is greater for the larger helix angles, due probably to friction on the side of the cut because of the end thrust or drag. The general tendency of the exponents is to drop with increasing rake angles above 10 to 12 deg, and to rise with increasing helix angles above 20 deg. This effect is more noticeable with the depth exponents than with the feed exponents, but is not great in any case, except for the 30- and 40-deg helix angle with groove cuts which, undoubtedly, is caused by side friction. The same effect also is noticed in the values of the constants.

To compare the actual energy and horsepower values as affected by a change in rake and helix angles for the various cuts, Fig. 3 is shown. The difference between up and down milling is very noticeable. Increasing the rake angle is found to decrease energy values, first rapidly up to about 20 deg, and then more slowly. This probably would continue to an optimum angle, such as that formed by the built-up edge on the tool. It probably would be less pronounced for longer chips where the built-up edge would be more effective. The effect of a helix angle for this material appears to be quite favorable, but is consistently upward for groove cutting.

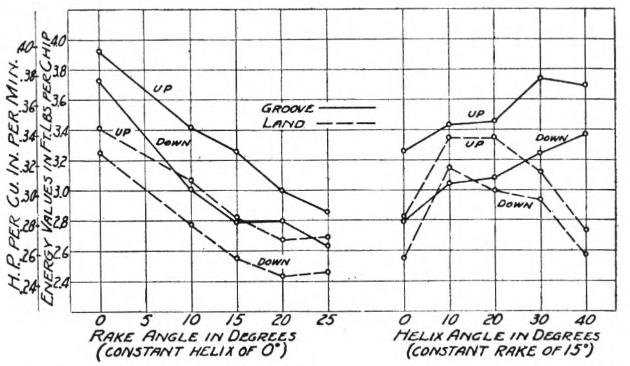

FIG. 3 EFFECT OF VARIOUS RAKE AND HELIX ANGLES OF THE CUTTER ON THE POWER AND ENERGY VALUES WHEN MILLING FREE-CUTTING BRASS

(Cuts were taken up and down both in a groove and on a land as indicated. Data taken from Tables 2 and 3.)

Tests on S.A.E. 1020 Steel (Bar EF). The complete series of tests as presented for the free-cutting brass were duplicated on the S.A.E. 1020 steel. Table 4 summarizes the formulas and the values in horsepower per cubic inch per minute. It is apparent that for most cuts in steel, a different formula is found for up and down cutting. This differs from the results of the tests on brass.

Again, to compare exponents and constants as a function of tool angles, Fig. 4 has been prepared. This figure is for steel, as Fig. 2 is for brass. The effect on the exponents of variable rake angles appears slight, but greater, with a less definite tendency indicated, when the helix angle is varied. The depth exponent appears to drop off with the greatest rake angle. The constant varies over a wide range as the rake angle is changed, being reduced considerably as the rake angle is increased from 0 to 25 deg. The constant of the equation appears to be little influenced by the value of helix angle up to and including 30 deg, but increases sharply for groove cutting when the helix angle is 40 deg. Again, it should be pointed out that a direct comparison of either the exponents or constants with rake angles cannot be made, inasmuch as both the exponents and constants of the equations vary.

The net effect on energy values in foot-pounds per chip and on the values of horsepower per cubic inch of metal removed per minute is shown for variable rake angles and variable helix angles in Fig. 5. The energy values for the 0-deg rake tool are more than 100 per cent greater than those for the 25-deg rake tool. The effect of the helix angle, however, appears to be negligible, except as affected by side friction in groove cutting.

TABLE 4 RESULTS OF TESTS WHEN MILLING S.A.E. 1020 STEEL, BAR *EF*

(An emulsion of 1 part soluble oil to 50 parts water, cutting fluid No. 3, was used. The width of the cutter was 0.300 in.)

Tool angles, deg Helix	Rake	Method of cutting	Formulas, energy *E*, in foot-pounds per chip Groove	Land	Values of *C* Groove	Land	Horsepower per cu in. per min formulas Groove	Land	Horsepower per cu[a] in. per min Groove	Land
0	0	Up	$Cwf^{0.77}d^{0.87}$	$Cwf^{0.65}d^{0.91}$	15000	9040	$\frac{C}{33000\,f^{0.23}d^{0.13}}$	$\frac{C}{33000\,f^{0.35}d^{0.09}}$	1.77	1.678
		Down	$Cwf^{0.79}d^{0.92}$		17080	8680	$\frac{C}{33000\,f^{0.21}d^{0.08}}$		1.638	1.617
0	10	Up	$Cwf^{0.75}d^{0.92}$	$Cwf^{0.70}d^{0.89}$	10860	7300	$\frac{C}{33000 f^{0.25}d^{0.08}}$	$\frac{C}{33000\,f^{0.30}d^{0.11}}$	1.253	1.132
		Down	....	$Cwf^{0.70}d^{0.93}$	8670	7350	....	$\frac{C}{33000\,f^{0.30}d^{0.07}}$	1.000	1.041
0	15	Up	$Cwf^{0.74}d^{0.89}$	$Cwf^{0.71}d^{0.89}$	8250	6340	$\frac{C}{33000\,f^{0.26}d^{0.11}}$	$\frac{C}{33000\,f^{0.29}d^{0.11}}$	1.070	0.940
		Down	$Cwf^{0.74}d^{0.91}$	$Cwf^{0.71}d^{0.91}$	7250	5340	$\frac{C}{33000\,f^{0.26}d^{0.09}}$	$\frac{C}{33000\,f^{0.29}d^{0.09}}$	0.898	0.757
0	20	Up	$Cwf^{0.74}d^{0.91}$	$Cwf^{0.71}d^{0.92}$	7380	5600	$\frac{C}{33000\,f^{0.26}d^{0.09}}$	$\frac{C}{33000\,f^{0.29}d^{0.08}}$	0.910	0.778
		Down			7280	4870			0.798	0.677
0	25	Up	$Cwf^{0.76}d^{0.87}$	$Cwf^{0.71}d^{0.86}$	6450	4570	$\frac{C}{33000\,f^{0.24}d^{0.13}}$	$\frac{C}{33000\,f^{0.29}d^{0.14}}$	0.798	0.727
		Down			5450	4130			0.677	0.657
10	15	Up	$Cwf^{0.78}d^{0.90}$	$Cwf^{0.74}d^{0.93}$	10360	8330	$\frac{C}{33000\,f^{0.22}d^{0.10}}$	$\frac{C}{33000\,f^{0.26}d^{0.03}}$	1.091	0.990
		Down	$Cwf^{0.79}d^{0.95}$		10490	7060	$\frac{C}{33000\,f^{0.21}d^{0.05}}$	....	0.940	0.838
20	15	Up	$Cwf^{0.76}d^{0.87}$	$Cwf^{0.72}d^{0.83}$	9770	5750	$\frac{C}{33000\,f^{0.24}d^{0.13}}$	$\frac{C}{33000\,f^{0.28}d^{0.17}}$	1.201	0.940
		Down	$Cwf^{0.76}d^{0.95}$	$Cwf^{0.73}d^{0.94}$	10000	6430	$\frac{C}{33000\,f^{0.24}d^{0.05}}$	$\frac{C}{33000\,f^{0.27}d^{0.06}}$	1.030	0.778
30	15	Up	$Cwf^{0.70}d^{0.87}$	$Cwf^{0.67}d^{0.84}$	7520	4970	$\frac{C}{33000\,f^{0.30}d^{0.13}}$	$\frac{C}{33000\,f^{0.33}d^{0.18}}$	1.233	1.000
		Down	$Cwf^{0.72}d^{0.98}$	$Cwf^{0.69}d^{0.86}$	10310	4350	$\frac{C}{33000\,f^{0.28}d^{0.02}}$	$\frac{C}{33000\,f^{0.31}d^{0.14}}$	1.192	0.758
40	15	Up	$Cwf^{0.79}d^{1.04}$	$Cwf^{0.69}d^{0.85}$	20200	5270	$\frac{Cd^{0.04}}{33000\,f^{0.21}}$	$\frac{C}{33000\,f^{0.31}d^{0.16}}$	1.465	0.940
		Down	$Cwf^{0.79}d^{1.07}$	$Cwf^{0.69}d^{0.91}$	19900	5450	$\frac{Cd^{0.07}}{33000\,f^{0.21}}$	$\frac{C}{33000\,f^{0.31}d^{0.09}}$	1.344	0.849

[a] 0.010 in. feed, 0.100 in. depth, 0.300 in. width.

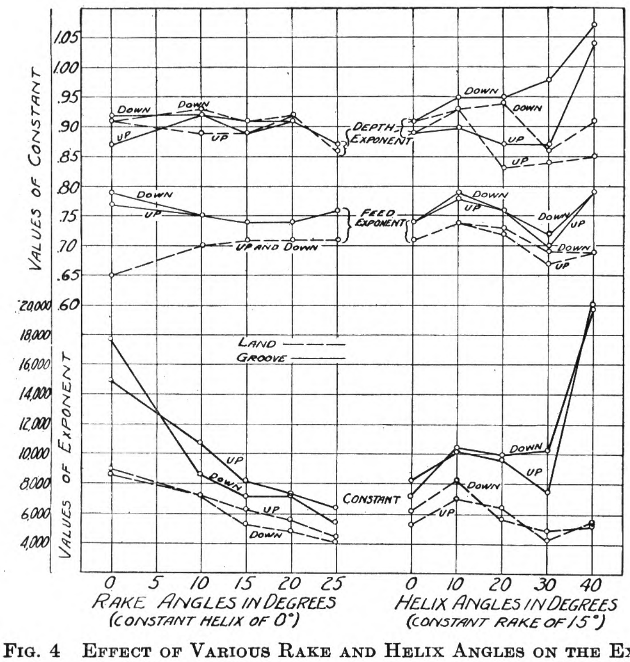

FIG. 4 EFFECT OF VARIOUS RAKE AND HELIX ANGLES ON THE EXPONENTS OF FEED AND DEPTH AND THE CONSTANT IN THE MILLING-ENERGY EQUATION WHEN MILLING S.A.E. 1020 STEEL, BAR *EF*

(Milling cuts were made up and down and on a land and in a groove as indicated. Data taken from Table 4.)

Tests on S.A.E. 1112 Steel (Bar MF). The results of the milling tests on this steel are summarized in Table 5, in which are given the formulas obtained, the constants for the formulas, and the horsepower per cubic inch cut per minute for a specific cut for both groove and land cutting, as well as for up and down cutting. Fig. 6, similar to Figs. 2 and 4, shows the exponents and constants for the free-cutting steel as a function of the tool angles. It is seen that the values of the constants are reduced appreciably with an increase in rake angle up to 10 deg, but only slightly for greater values of rake angle. There does not appear to be a great difference in general characteristics between the curves for land and groove cutting. When the constants are plotted over the helix angles, however, there is shown to be a general and gradual increase in value, although a minimum value is reached in nearly all cases for the helix angle of 10 deg. The values of the depth exponent appear to be little affected by an increase in rake angle until an angle of 20 to 25 deg is reached, when there is a slight falling off in the curves. The feed exponent, on the other hand, seems to increase rather generally for rake angles above 10 deg. The depth exponent appears to be influenced but slightly for helix angles up to and including 30 deg, while the feed exponent seems to be increased uniformly for angles above 10 deg, for which minimum values occur.

A comparison of energy values in foot-pounds per chip and horsepower per cubic inch of metal removed per minute, as indicated, are expressed as a function of the variable rake angle and

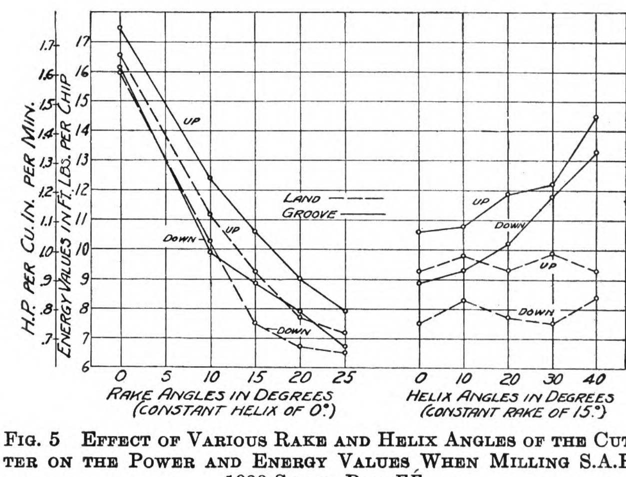

FIG. 5 EFFECT OF VARIOUS RAKE AND HELIX ANGLES OF THE CUTTER ON THE POWER AND ENERGY VALUES WHEN MILLING S.A.E. 1020 STEEL, BAR *EF*

(Cuts were taken up and down both in a groove and on a land as indicated Data taken from Table 4.)

TABLE 5 RESULTS OF TESTS WHEN MILLING S.A.E. 1112 STEEL, BAR *MF*

(An emulsion of 1 part soluble oil to 50 parts water, cutting fluid No. 3, was used. The width of the cutter was 0.300 in.)

Tool angles, deg Helix	Rake	Method of cutting	Formulas, energy E, in foot-pounds per chip Groove	Land	Values of constant Groove	Land	Horsepower per cu in. per min, formulas Groove	Land	Hp per cu in. per min Groove	Hp per cu in. per min Land
−0	0	Up	$Cwf^{0.74}d^{0.90}$	$Cwf^{0.73}d^{0.86}$	10180	8350	$\frac{C}{33000\ f^{0.26}d^{0.10}}$	$\frac{C}{33000\ f^{0.27}d^{0.14}}$	1.293	1.212
		Down		$Cwf^{0.69}d^{0.86}$	8440	5670		$\frac{C}{33000\ f^{0.31}d^{0.14}}$	1.071	0.99
0	10	Up	$Cwf^{0.71}d^{0.88}$	$Cwf^{0.70}d^{0.90}$	6360	6050	$\frac{C}{33000\ f^{0.29}d^{0.12}}$	$\frac{C}{33000\ f^{0.30}d^{0.10}}$	0.959	0.93
		Down		$Cwf^{0.68}d^{0.88}$	5270	4440		$\frac{C}{33000\ f^{0.32}d^{0.12}}$	0.798	0.778
0	15	Up	$Cwf^{0.77}d^{0.85}$	$Cwf^{0.74}d^{0.90}$	7550	6560	$\frac{C}{33000\ f^{0.23}d^{0.15}}$	$\frac{C}{33000\ f^{0.26}d^{0.10}}$	0.930	0.848
		Down	$Cwf^{0.75}d^{0.85}$	$Cwf^{0.71}d^{0.85}$	5710	4570	$\frac{C}{33000\ f^{0.25}d^{0.15}}$	$\frac{C}{33000\ f^{0.29}d^{0.15}}$	0.768	0.725
0	20	Up	$Cwf^{0.79}d^{0.83}$	$Cwf^{0.72}d^{0.87}$	6450	4920	$\frac{C}{33000\ f^{0.21}d^{0.17}}$	$\frac{C}{33000\ f^{0.28}d^{0.13}}$	0.758	0.73
		Down	$Cwf^{0.76}d^{0.86}$	$Cwf^{0.69}d^{0.87}$	5360	3910	$\frac{C}{33000\ f^{0.24}d^{0.14}}$	$\frac{C}{33000\ f^{0.31}d^{0.13}}$	0.677	0.664
0	25	Up	$Cwf^{0.80}d^{0.82}$	$Cwf^{0.75}d^{0.84}$	6250	5080	$\frac{C}{33000\ f^{0.20}d^{0.18}}$	$\frac{C}{33000\ f^{0.25}d^{0.16}}$	0.718	0.705
		Down		$Cwf^{0.70}d^{0.84}$	5710	3710		$\frac{C}{33000\ f^{0.30}d^{0.16}}$	0.657	0.643
10	15	Up	$Cwf^{0.74}d^{0.85}$	$Cwf^{0.72}d^{0.82}$	6340	4960	$\frac{C}{33000\ f^{0.26}d^{0.15}}$	$\frac{C}{33000\ f^{0.28}d^{0.18}}$	0.899	0.828
		Down	$Cwf^{0.76}d^{0.85}$	$Cwf^{0.69}d^{0.82}$	5940	3810	$\frac{C}{33000\ f^{0.24}d^{0.15}}$	$\frac{C}{33000\ f^{0.31}d^{0.18}}$	0.768	0.726
20	15	Up	$Cwf^{0.81}d^{0.86}$	$Cwf^{0.78}d^{0.90}$	9100	7740	$\frac{C}{33000\ f^{0.19}d^{0.14}}$	$\frac{C}{33000\ f^{0.22}d^{0.10}}$	0.910	0.815
		Down	....	$Cwf^{0.70}d^{0.85}$	8180	4370	....	$\frac{C}{33000\ f^{0.30}d^{0.15}}$	0.819	0.74
30	15	Up	$Cwf^{0.79}d^{0.87}$	$Cwf^{0.76}d^{0.83}$	8800	6470	$\frac{C}{33000\ f^{0.21}d^{0.13}}$	$\frac{C}{33000\ f^{0.24}d^{0.17}}$	0.960	0.877
		Down		$Cwf^{0.73}d^{0.83}$	7770	4930		$\frac{C}{33000\ f^{0.27}d^{0.17}}$	0.839	0.77
40	15	Up	$Cwf^{0.82}d^{0.95}$	$Cwf^{0.76}d^{0.88}$	12780	7170	$\frac{C}{33000\ f^{0.18}d^{0.05}}$	$\frac{C}{33000\ f^{0.24}d^{0.12}}$	1.051	0.87
		Down			12100	6520			0.940	0.788

variable helix angle in Fig. 7 for both groove and land cutting. The curves for groove cutting are higher than the corresponding curves for land cutting. Also, the curves for down cutting are uniformly lower than those for up cutting. The energy or power values are seen to be maximum for 0-deg rake angle, and to fall off uniformly, as the rake angle is increased, to almost a minimum for a rake angle of 25 deg. It appears that for greater values of rake angle there would be no further appreciable reduction in energy values. The energy values as a function of the helix angle are shown to be practically constant when cutting on the land, indicating that the helix angle does not lead to efficiency from a power point of view. When cutting in a groove, the energy values for both up and down cutting increase for

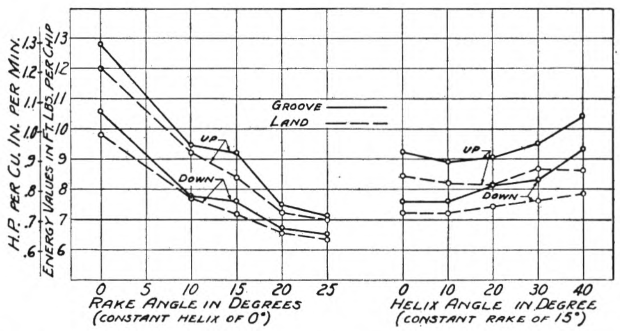

FIG. 7 EFFECT OF VARIOUS RAKE AND HELIX ANGLES OF THE CUTTER ON THE POWER AND ENERGY VALUES WHEN MILLING S.A.E. 1112 STEEL, BAR *MF*

(Cuts were taken up and down both in a groove and on a land as indicated. Data taken from Table 5.)

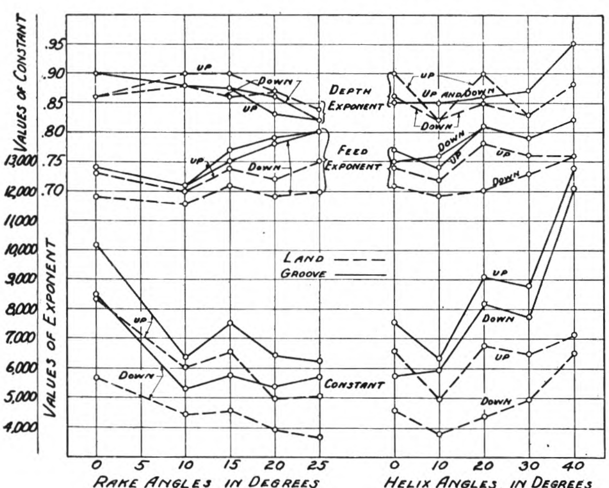

FIG. 6 EFFECT OF VARIOUS RAKE AND HELIX ANGLES ON THE EXPONENTS OF DEPTH AND FEED AND THE CONSTANT IN THE MILLING-ENERGY EQUATION WHEN MILLING S.A.E. 1112 STEEL, BAR *MF*

(Cuts were made up and down and on a land and in a groove as indicated. Data taken from Table 5.)

values of helix angle above 20 deg, due undoubtedly to the drag of the chip on one side of the groove.

Tests on S.A.E. 3250 Steel (*Bar A-7*). The results of the experiments in milling S.A.E. 3250 steel are shown in Table 6, in which are given the formulas for energy per chip, the values of the constants for the energy equation, and formulas for horsepower per cubic inch per minute, together with specific values of horsepower per cubic inch of metal removed per minute when milling up and down in a groove and on a land. An analysis of the constants and exponents of the formulas as influenced by variable rake or variable helix angles is shown in Fig. 8.

The depth exponent appears to be nearly constant over a wide range of rake angles and also over a wide range of helix angles. For the largest rake and helix angles, however, the exponent for groove milling is high. The feed exponent seems to fall off uniformly and appreciably as the rake angle is increased, but remains practically constant over the whole range of helix angles used. This holds for both up and down milling on a land or in a groove. The constants of the formula fall off quite uniformly

TABLE 6 RESULTS OF TESTS WHEN MILLING S.A.E. 3250 STEEL, BAR *A*-7

(An emulsion of 1 part soluble oil to 50 parts water, cutting fluid No. 3, was used. The width of the cutter was 0.300 in.)

Tool angles, deg Helix	Tool angles, deg Rake	Method of cutting	Formulas, energy *E*, in foot-pounds per chip: Groove	Formulas, energy *E*, in foot-pounds per chip: Land	Values of constant: Groove	Values of constant: Land	Horsepower per cu in. per min, formulas: Groove	Horsepower per cu in. per min, formulas: Land	Horsepower per cu in. per min: Groove	Horsepower per cu in. per min: Land
0	0	Up	$Cwf^{0.77}d^{0.87}$	$Cwf^{0.82}d^{0.85}$	11400	13360	$\frac{C}{33000\, f^{0.23}d^{0.13}}$	$\frac{C}{33000\, f^{0.18}d^{0.15}}$	1.343	1.313
		Down	$Cwf^{0.77}d^{0.95}$	$Cwf^{0.84}d^{0.85}$	14880	13630	$\frac{C}{33000\, f^{0.23}d^{0.05}}$	$\frac{C}{33000\, f^{0.16}d^{0.15}}$	1.455	1.212
0	10	Up Down	$Cwf^{0.75}d^{0.91}$	$Cwf^{0.78}d^{0.90}$	10750	10500	$\frac{C}{33000\, f^{0.25}d^{0.09}}$	$\frac{C}{33000\, f^{0.22}d^{0.10}}$	1.263	1.11
0	15	Up Down	$Cwf^{0.74}d^{0.92}$	$Cwf^{0.76}d^{0.88}$	9720	8860	$\frac{C}{33000\, f^{0.26}d^{0.08}}$	$\frac{C}{33000\, f^{0.24}d^{0.12}}$	1.172	1.07
0	20	Up Down	$Cwf^{0.71}d^{0.87}$	$Cwf^{0.73}d^{0.87}$	6780	6880	$\frac{C}{33000\, f^{0.29}d^{0.13}}$	$\frac{C}{33000\, f^{0.27}d^{0.13}}$	1.05	0.939
0	25	Up Down	$Cwf^{0.68}d^{0.95}$	$Cwf^{0.67}d^{0.89}$	7040	5280	$\frac{C}{33000\, f^{0.32}d^{0.05}}$	$\frac{C}{33000\, f^{0.33}d^{0.11}}$	1.041	0.99
10	15	Up Down	$Cwf^{0.75}d^{0.88}$	$Cwf^{0.75}d^{0.88}$	9880	8480	$\frac{C}{33000\, f^{0.25}d^{0.12}}$	$\frac{C}{33000\, f^{0.25}d^{0.12}}$	1.242	1.07
20	15	Up Down	$Cwf^{0.72}d^{0.86}$	$Cwf^{0.76}d^{0.90}$	8000 7340	9530	$\frac{C}{33000\, f^{0.28}d^{0.14}}$	$\frac{C}{33000\, f^{0.24}d^{0.10}}$	1.212 1.111	1.11
30	15	Up Down	$Cwf^{0.72}d^{0.86}$	$Cwf^{0.76}d^{0.93}$	10000	10670	$\frac{C}{33000\, f^{0.28}d^{0.14}}$	$\frac{C}{33000\, f^{0.24}d^{0.07}}$	1.516	1.15
40	15	Up Down	$Cwf^{0.74}d^{0.98}$	$Cwf^{0.78}d^{0.93}$	18450	12250	$\frac{C}{33000\, f^{0.26}d^{0.02}}$	$\frac{C}{33000\, f^{0.22}d^{0.07}}$	1.940	1.212

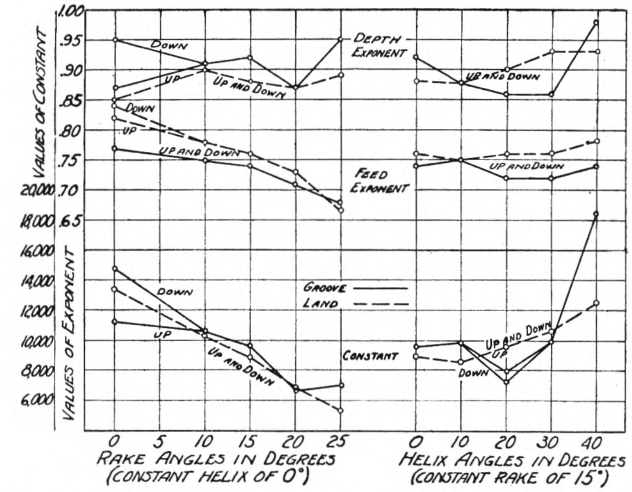

Fig. 8 Effect of Various Rake and Helix Angles on the Exponents of Feed and Depth and the Constant in the Milling-Energy Equation When Milling S.A.E. 3250 Steel, Bar *A*-7

(Milling cuts were made up and down and on a land and in a groove as indicated. Data taken from Table 6.)

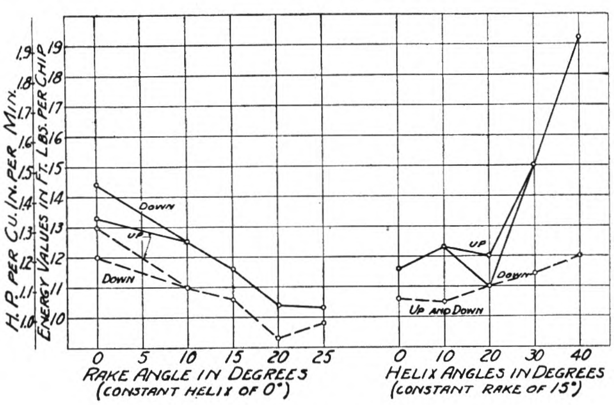

Fig. 9 Effect of Various Rake and Helix Angles of the Cutter on the Power and Energy Values When Milling S.A.E. 3250 Steel, Bar *A*-7

(Cuts were taken up and down both in a groove and on a land as indicated. Data taken from Table 6.)

for all cutting conditions as the rake angle is increased from 0 to 25 deg. The constant slightly increases, however, for up and down land milling for helix angles above 10 deg. For groove milling, the constant appears to be quite erratic, particularly for helix angles of 20 and 40 deg; otherwise they are unaffected by the helix angle.

The energy and power values appear to fall off for both up and down milling, as well as land and groove milling, as the rake angle is increased, as shown in Fig. 9. In up and down milling on a land, the energy values for the 0- and 10-deg. helix angles are the same, but they become greater as the helix angle is increased above 10 deg. The energy values for groove milling both up and down, as a function of helix angles, are extremely out of order for helix angles above 20 deg, as shown by the unusual lines of the curve. Again it is noticed that the power values for groove cutting are consistently above those for land cutting as would be expected.

Discussion of Single-Point-Cutter Data

In beginning the discussion, the authors wish to point out and make clear a fact which has been found repeatedly: if the material is changed, the constants and exponents in the formulas, as well as the energy values, are changed. Also, if the cutter is changed, the constants, exponents, and energy values are changed.

With this in mind, the object of this particular study in the Elements of Milling has been to compare various tool angles, various sizes, and methods of cut on only enough metals to be reasonably sure that the conclusions reached will hold in general for most metals.

After a complete series of tests was run on the free-cutting brass and formulas were determined, it was discovered that the formula for the 0-deg helix, 15-deg rake tool did not check the corresponding formula given in the first report. The two equations were as follows:

From the first paper:

$$E = Cwf^{0.76}d^{0.96}$$

From the second test:

$$E = Cwf^{0.85}d^{0.96}$$

A certain experimental error, of course, was to be expected. A difference in the feed exponent from 0.76 to 0.85, however, hardly could be considered as an experimental error. Also, the energy values and constants varied somewhat. Repeated checks confirmed the accuracy of the second series as represented by the second formula. The cutters used were the same brand of high-speed steel, and equal care was taken in each case in the grinding. Further check tests were run on the piece of stock used in the first paper. These tests were found to give a feed exponent of 0.77, comparing favorably with 0.76 of the first paper. Obviously, the material used in the experiments for this second paper had some different physical characteristics from the specimens used in the first paper, showing that the material at different parts of the same bar was not uniform.

Recently, a series of comparative tests was run on cold-drawn brass rod about $^7/_{16}$ in. in diameter, which showed machining differences in different lengths depending upon which end of the billet was machined. Differences in grain size, in heat treatment, or small differences in composition all affect the values obtained. It is surprising, however, to note the uniformity obtained in the value of the exponents of feed and depth in the energy equation when cutting different types of steels as explained below. It previously has been found that, in determining equations for torque and thrust in drilling steel under a wide variety of cutting conditions and covering many types of steels, the same equations have been developed repeatedly.

In order to arrive at the general tendencies introduced by the variables of the cutters and methods of cutting steel, the results in the three steels tested have been correlated. Of the three steels tested, S.A.E. 1112 steel gives energy values slightly below the average for steel, S.A.E. 1020 gives energy values presumably fairly average for steel in general, while S.A.E. 3250 gives power values slightly above the average steel. Steels S.A.E. 1020 and S.A.E. 1112 give lower values of energy when cutting down than when cutting up, while S.A.E. 3250 shows very little difference in energy values for these two types of cuts. By averaging all of the data for these three steels, it is believed a general picture of the effect of angles, methods of cutting, etc. is obtained for an average steel. This picture may be compared with that for each individual steel.

Variation of Constant in Energy Equation. The average values of the constants in the energy formula for the three steels are shown plotted over the rake angle or hook in Fig. 10. The constants for cutting down are considerably lower in all cases, except that for 0-deg rake, than when cutting up. The constant

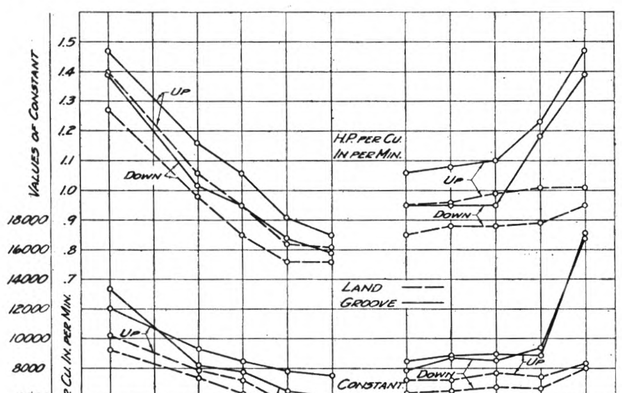

FIG. 10 AVERAGE VALUES OF THE CONSTANTS IN THE ENERGY FORMULA FOR THE THREE STEELS S.A.E. 1112, 1020, AND 3250, FOR UP AND DOWN MILLING AND FOR LAND AND GROOVE CUTTING, PLOTTED OVER THE RAKE ANGLE AND HELIX ANGLE IN DEGREES AS THE VARIABLES

(The horsepower-per-cubic-inch-per-minute values as averaged for the three steels also are shown. The cut was 0.300 in. wide, 0.100 in. deep, and the feed per cut was 0.010 in.)

on an average for the 25-deg rake angle is about 55 per cent of that for the 0-deg rake angle. There is a tendency, however, for the constant curves to become horizontal for the greater values of rake, which agrees in general with similar experiments conducted with an end-cutting planer tool in which the back rake was varied. It also is seen that the constants for land cutting are considerably lower than the corresponding values for groove cutting.

The constants when plotted over side rake or helix angles in Fig. 10 are seen to remain practically the same, with perhaps a slight increase, for all side-rake angles up to and including 30 deg. There is a marked increase in the value of the constant for the 40-deg side-rake angle, due presumably to excessive metallic distortion of the chip. This same condition was found when studying side-rake angles on end-cutting planer tools. The constants are higher for groove cutting due undoubtedly to increased friction.

Variation of Horsepower per Cubic Inch per Minute. Average values of horsepower per cubic inch per minute when taking a

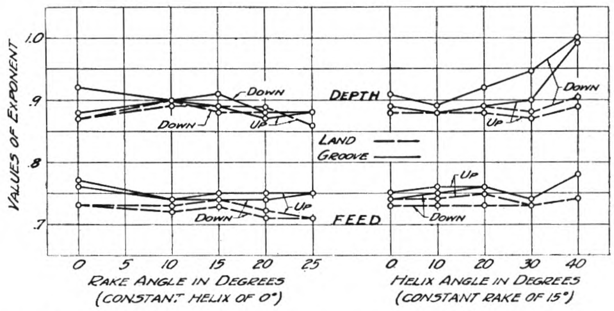

FIG. 11 AVERAGE VALUES OF THE EXPONENTS OF FEED AND DEPTH FOR THE FORMULAS OF THE THREE STEELS ARE SHOWN PLOTTED OVER THE RAKE ANGLE AND HELIX ANGLE AS VARIABLES

cut 0.300 in. wide, 0.100 in. deep, with a feed per cut 0.010 in., when using a tool 0.300 in. wide with various values of side rake and helix, are shown in Fig. 10. The power per cubic inch per minute is seen to fall off in almost direct proportion to the increase in rake up to and including 20 deg. There appears to be little increased efficiency due to a further increase of the rake angle to 25 deg. The curve for groove cutting up is well above that for land cutting up; also, that for groove cutting down is above that for land cutting down. However, the curves of horsepower per cubic inch per minute for groove cutting down and land cutting up show about the same values. The values of horsepower per cubic inch per minute, as an average, show but a slight increase for the 15-deg rake tool when the helix angle is increased from 0 to 20 deg. For higher helix angles, however, when cutting in a groove, the power increases rapidly due to side friction, but for land cutting the same tendency as indicated for lower helix angles is maintained.

Variation of Exponents of Depth and Feed. With the average of the exponents for feed and depth plotted over the variable rake or helix angle in Fig. 11, it is seen that the exponent for depth when cutting down averages higher than when cutting up; also, that the exponent of the feed when cutting down averages lower than when cutting up. Except for high depth exponents for high helix angles in groove milling, all exponents are practically constant, with the following slight but consistent tendencies:

1 The depth exponents fall off slightly with high rake angles, and rise slightly for high helix angles when milling in a groove. But when the milling is done on a land, they are practically constant.

2 The feed exponents remain practically constant as the rake angle increases, although a slight decrease is apparent for land cutting. The feed exponents remain practically constant as the helix increases, except for extreme values of helix in groove cutting where the exponent is high.

Comments on Finish Produced

No great difference in finish for a given type of cut was noticed due to changing tool angles. The nature of the tests excluded any possibility of studying cutter life. An investigation of finish, cutter life, and pressures is planned on apparatus already set up.

The difference in finish on up and down milling, however, was quite apparent. The finish on up milling was, in general, smooth and shiny or glassy in appearance. Very often it showed distinct feed marks and scratches which were sometimes quite deep. The chips had a tendency to cling to the cutting edge, and, if left on, would be dragged under the cutter on the next cut, marring the surface. On down cutting, the finish was smooth but duller with a satin-like appearance. Feed marks were usually not visible, but on some materials the surface produced was rougher. Chips had little tendency to cling to the tool and did no harm, as they would be knocked off at the start of the cut, whereas the finish is made at the end of the cut.

Chatter marks were more common when cutting up than when cutting down. As far as chatter and finish were concerned it was usually possible when milling down to increase the feed considerably over that when milling up.

Shape of Chip. Chips taken by the two different methods often showed distinct characteristics. When down milling the screw-stock steel S.A.E. 1112, for example, very tightly rolled chips were produced. On up milling, the chips were straight or flat. These characteristics were observed for all variable back-rake-angle tools, but when the tools of increasing helix angle were used the chip forms approached each other, the down-milled chips becoming less tightly rolled and the up-milled chips tending to curl. Chips from the 30- and 40-deg helix-angle cutters were almost identical for both methods of cutting.

When down milling the chromium-nickel steel S.A.E. 3250, the chips were flat, whereas tightly rolled chips were obtained on up milling. Again these differences disappeared with the higher helix-angle tools.

In general, it may be said that the down-milled chips are rolled more tightly than the up-milled chips, the exceptions occurring, in most cases, with those metals that show little or no difference in energy values for the two methods.

Built-Up edge. Undoubtedly, chip form is influenced by the built-up edge. In general, a better developed built-up edge forms on milling down. It seems reasonable to expect that a large, well-developed edge will coil the chip, while a small poorly developed edge might allow a flatter chip to be formed. This is borne out by observing the chips. In the case of the S.A.E. 3250 steel, a well-developed edge was formed, the chip was flat in down milling, but was coiled in up milling. This action was contrary to that generally observed. In fact, exceptionally large edges were noticed for this steel, and the surface left by down milling was rougher than usual, due to the roughening effect of the built-up edge.

Because of this roughening effect of the built-up edge, it is possible to detect from the chip the history of its formation. The under face of the chips is highly burnished for a distance, depending upon the metal, up to $^1/_{16}$ in., which corresponds to $^1/_8$ to $^3/_{16}$ in. actual length of cut. Then the surface loses its luster and reaches a maximum degree of roughness in an added $^1/_{16}$ in. of length. Well-developed edges are often found with depths of cut of 0.025 in. or less, corresponding to about $^1/_4$ in. length of cut with the 3.5 in. in diameter cutter, but as a rule somewhat longer length of cut is necessary for the fully developed edge. After being built up, further increase in depth of cut does not change it materially. Also, less length of cut to form this edge is required for down milling than for up milling, the amount of this difference depending upon the metal and apparently influenced by tool angles, especially back rake. This difference usually decreases as the back-rake angle increases.

The burnished length on the chip generally increases somewhat as the back rake is increased, but some of this increase may be due to less compacting of the metal in the chip. That is, the chips are longer for the same size of cut. The edge is usually thicker for the low back-rake angles, but does not seem to require much, if any, greater length of cut for its formation. Changing the side rake or helix angle does not seem to change the formation of built-up edge to any extent, but does affect the rolling of the chip.

A more detailed and scientific study of the important factor of built-up edge has not been undertaken because of the nature of the tests. The authors expect to do this in the course of tests for the next paper of this series when a standard milling machine will be used and cuts will be taken as the work is fed.

Analysis of Some Milling Data Obtained Under Commercial Conditions

In 1929, a thesis, "Milling Cutter Efficiency" (Experiment Station Project No. 85), was presented at the Ohio State University by four men, Otto W. Winter, Charles E. Beard, Howard W. Allison, and Robert J. Duerler. The work, involving an expenditure of several thousand dollars, was sponsored by the Cincinnati Milling Machine Company. A total of about 24,000 cuts were run.

In discussing "Elements of Milling,"[4] Mr. Winter[5] suggested that a comparison of these data with those given in his report might be of distinct value.

In the Ohio State experiments, standard coarse-tooth high-speed-steel cutters were used having spiral and under-cut teeth. A wide range of feeds and depths of cut were covered. The materials cut consisted of cast iron and S.A.E. 3125 to 3140 steels. Power data were presented as calibrated net horsepower at the cutter for specified cuts, feeds, cutters, and material. All cuts were taken in the conventional manner—that is, up cutting. An emulsion consisting of 1 part Sunoco soluble oil to 20 parts water was used on the steel, but the cast iron was cut dry.

Because the data were taken with standard cutters on a standard milling machine and because such a wealth of data is available covering a range of feeds, depths of cut, and widths that were not possible with a small pendulum tester, the authors believed that much could be learned and many questions answered by recomputing these data and comparing the results with those of the paper.

Apparatus. The apparatus used in the Ohio State experiments consisted of a no. 2M Cincinnati plain miller of the latest (1929) inclosed, 7.5-hp-motor type, equipped with a wattmeter for registering gross power developed.

By means of a prony brake mounted on the spindle of the machine, efficiency curves of the motor and machine were determined for every speed and load variation. Power absorbed in the feed mechanism was neglected. This introduced a small variable error. Standard cutters of the shell-end, half-side, slotting, and slabbing types were used.

Procedure. The procedure of the tests was to take four cuts, from which the average power value was determined. If the data varied widely, additional cuts were taken. The dimensions of the cut, that is, the width and depth, were determined from the size and type of the cutter used, as indicated in Table 8. The feeds varied from $^1/_2$ to 20 in. per min.

As stated in the thesis, "Probably the greatest cause of power requirement variation, and certainly one that caused us much trouble, was the condition of the cutting edge. Throughout the

[4] Trans. A.S.M.E., vol. 54, 1932, paper no. RP-54-4.

[5] Cincinnati Milling Machine Company, Detroit, Mich.

course of the tests, we endeavored to run each of the four cuts, constituting a test, at varying periods of the life of the cutter between sharpenings." It follows that the data presented are not for the sharp condition, but rather for an average or partially dull condition of the tool.

Data. The data were presented as a series of curves, as illustrated in Fig. 13, for specified widths and depths of cut giving net horsepower (at the cutter) values plotted over feeds in inches per minute. A separate set of curves was presented for each combination of cutter and material.

In the following analysis by the authors, by depth of cut is meant the distance of metal cut measured in the plane of rotation of the cutter, and width of cut the distance measured on a line parallel to the axis of the cutter, and has nothing to do with the shape of the work or the position of the cut. This is a logical procedure and enables direct comparison of all data including that of the authors. It is usual to think of these dimensions as being reversed in a face-milling type of cut, and consequently the meaning of the designations d for depth and w for width, as shown in Fig. 12, should be kept in mind.

In Fig. 12 are shown five different types of cutters, each taking its typical cut. The principal object of this illustration is to indicate what is meant in each case by depth of cut d and the width of cut w. In the case of the plain milling cutters at A,

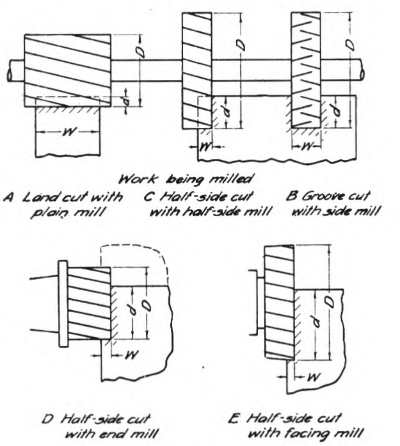

FIG. 12 ILLUSTRATIONS OF VARIOUS TYPES OF CUTS SHOWING A LAND CUT BEING MADE BY A PLAIN MILL, A GROOVE CUT BY A SIDE MILL, AND A HALF-SIDE CUT BY A HALF-SIDE, SHELL-END AND FACING MILL

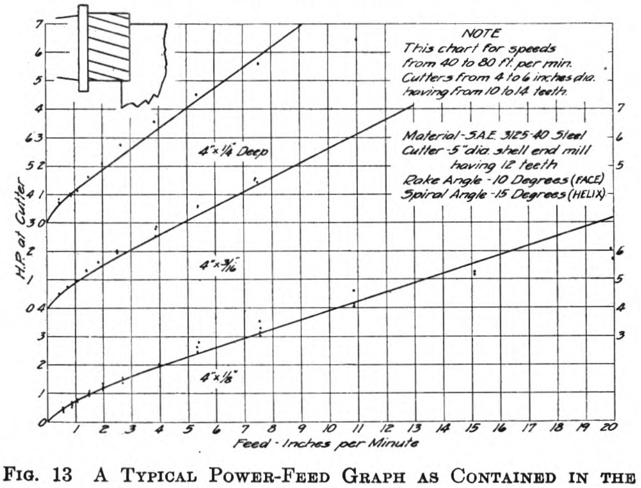

FIG. 13 A TYPICAL POWER-FEED GRAPH AS CONTAINED IN THE REPORT OF THE OHIO STATE EXPERIMENTS

(This shows the net horsepower at the cutter as determined by the wattmeter for various values of feed in inches per minute for three specific cuts as indicated.)

where the cut is being made on a land, the cutting teeth are in contact with the work only along the face of the land or on the periphery of the cutter. At B, in which a groove is being cut with a side or slotting mill, a cutting action occurs on the cylindrical periphery of the cutter and also on both ends or faces of the cutter. At C, in which a half-side mill is being used, the teeth are cutting on the periphery and on one end of the cutter. This same condition maintains with the shell-end mill at D and the facing mill at E.

From an analysis of the cuts A, B, and C, it might be expected that the half-side cutter would give energy values intermediate between the plain mill cut at A and the side or groove-cutting cutter at B, for the same size of cut. Data showing the relation of various factors in the energy equations are given in Table 8, and are covered under the discussion of the Ohio State Experimental data. It is found, however, that the net energy values at the cutter, as determined by subtracting the frictional power of the machine from the gross power input as measured by the wattmeter, give energy and power equations for all five cuts of the general type $E = Cwf^x d^y$, which corresponds with all energy equations determined on the single-tooth pendulum-type tester.

Computations of Ohio State Data. An example follows showing the method of computing energy values per chip as a function of feed and depth of cut from the net horsepower values obtained in the Ohio State tests.

The cutter in this case was a shell-end mill 5 in. in diameter, as illustrated in Fig. 13. It had 12 teeth having 10-deg rake

TABLE 7

(Values of energy in foot-pounds per chip for various combinations of feed, depth of cut, and width, determined in the Ohio State experiments. Average values of power shown by circles in Fig. 13 were used. An end mill having 12 teeth, 10-deg rake, and 15-deg helix was used in cutting S.A.E. 3125–3140 steels. An emulsion of 1 part oil to 20 parts water was used. Values are shown plotted in Fig. 14)

	Feed		1/8-in.-wide cut		3/16-in.-wide cut		1/4-in.-wide cut	
Depth in inches	In. per min	In. per tooth	Hp at cutter	Energy ft-lb per chip	Hp at cutter	Energy ft-lb per chip	Hp at cutter	Energy ft-lb per chip
2	0.5	0.00091	0.25	15	0.25	15	0.3	18
	0.75	0.00136	0.32	19	0.32	19	0.43	26
	1.0	0.0018	0.40	23.6	0.45	27	0.57	33
	1.4	0.00254	0.48	29	0.60	36	0.75	45
	1.9	0.00344	0.6	36	0.75	45	1.0	60
	2.6	0.0047	0.75	45	1.0	60	1.3	72
	3.9	0.0071	1.0	60	1.4	84	1.9	114
	5.5	0.010	1.25	75	1.9	114	2.45	147
	7.4	0.0137	1.56	94	2.4	144	3.2	192
	10.75	0.0195	2.25	135	3.5	210	4.5	270
	15.0	0.0272	3.1	186	4.7	282	6.2	370
	20.0	0.0363	4.16	250	6.2	370	...	...
3	0.5	0.00091	0.35	21	0.35	21	0.45	27
	0.75	0.00136	0.45	27	0.5	30	0.65	39
	1.0	0.0018	0.57	34	0.67	40	0.80	48
	1.4	0.00254	0.70	42	0.85	51	1.15	69
	1.9	0.00344	0.87	52	1.2	72	1.5	90
	2.6	0.0047	1.25	75	1.5	90	1.8	109
	3.9	0.0071	1.5	90	2.0	120	2.7	162
	5.5	0.010	1.9	114	2.7	162	3.5	210
	7.4	0.0137	2.5	150	3.6	216	4.6	278
	10.75	0.0195	3.4	204	5.0	300	6.4	384
	15.0	0.0272	4.4	264	6.8	407	...	...
	20.0	0.0363	6.0	360	...	...	...	...
4	0.5	0.00091	0.47	28	0.5	30	0.70	42
	0.75	0.00136	0.6	36	0.73	44	1.0	60
	1.0	0.0018	0.75	45	0.9	54	1.15	70
	1.4	0.00254	1.0	59	1.2	72	1.58	95
	1.9	0.00344	1.25	75	1.5	90	1.9	115
	2.6	0.0047	1.5	90	1.9	115	2.5	150
	3.9	0.0071	2.0	119	2.7	156	3.4	204
	5.5	0.010	2.5	150	3.5	210	4.5	270
	7.4	0.0134	3.2	192	4.55	274	5.6	336
	10.75	0.0195	4.3	258	6.0	360	...	...
	15.0	0.0272	5.5	330	...	...	...	...
	20.0	0.0365	7.2	432	...	...	...	...
5	0.5	0.00091	...	...	0.65	39	0.8	48
	0.75	0.00136	...	...	0.9	54	1.1	67
	1.0	0.0018	...	...	1.2	72	1.33	80
	1.4	0.00254	...	...	1.5	90	1.9	114
	1.9	0.00344	...	...	2.0	120	2.5	150
	2.6	0.0047	...	...	2.67	160	3.2	192
	3.9	0.0071	2.4	144	3.6	216	4.4	264
	5.5	0.010	3.0	180	4.5	270	5.6	335
	7.4	0.0134	3.8	228	5.7	340	6.7	400
	10.75	0.0195	5.0	300	7.5	450	...	...
	15.0	0.0272	6.2	372	...	...	...	...
	20.0	0.0365	8.0	480	...	...	...	...

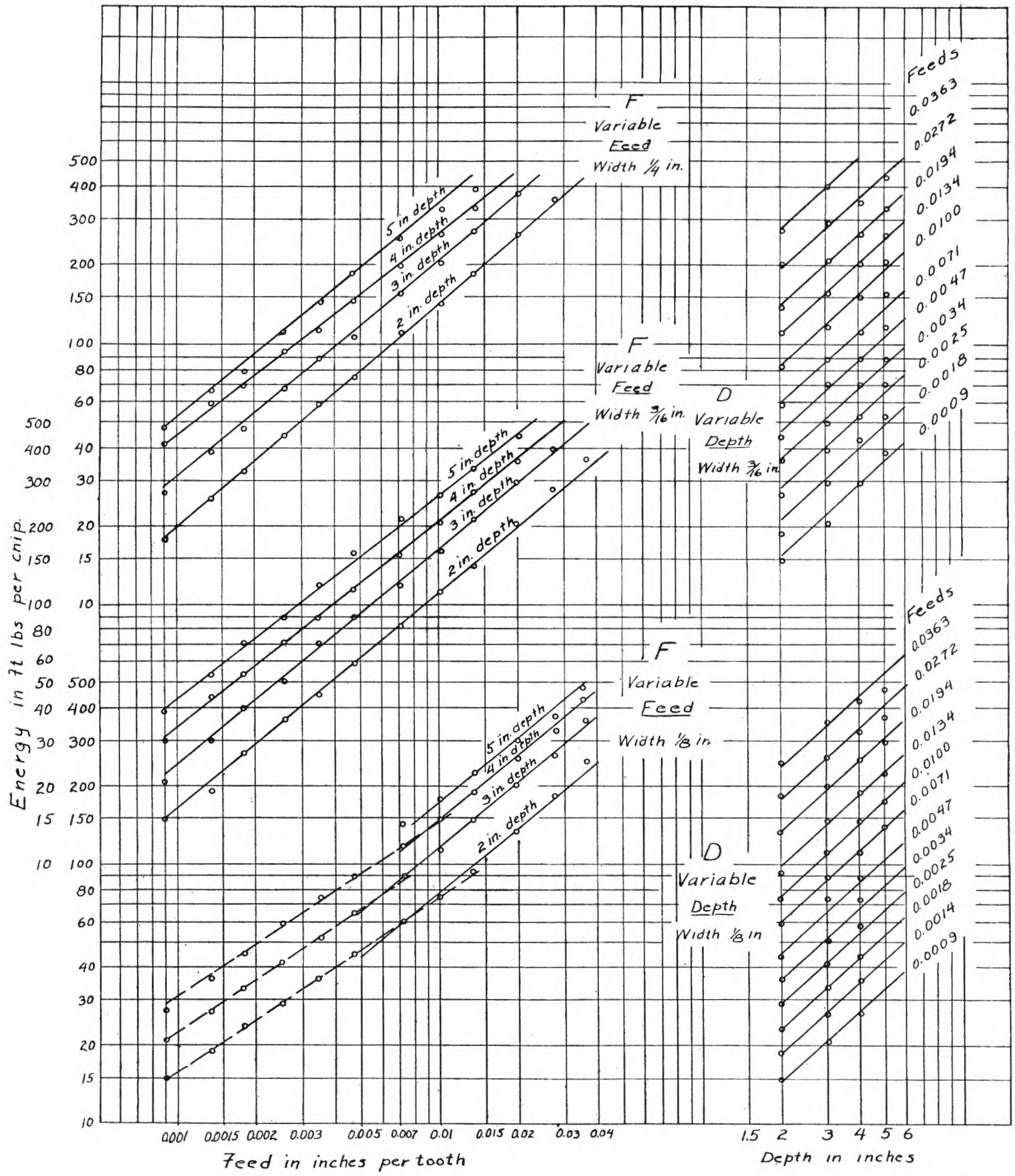

Fig. 14 Values of Energy as Computed From the Ohio State Experiments for the Shell-End Mill Having 12 Teeth, 10-Deg Rake, and 15-Deg Helix, Shown in Fig. 13

(Cutting S.A.E. 3125–3140 steels and plotted on log-log paper for various combinations of feed, depth of cut, and width of cut. An emulsion of 1 part soluble oil to 20 parts water was used. Energy values from Table 7. The general formula was obtained in which $E = 14{,}600\ wf^{0.83}d^{0.90}$.)

angle and 15-deg spiral or helix angle. It operated at 46 rpm, corresponding to a peripheral cutting speed of 60 ft per min.

The data cover tests on S.A.E. 3125, 3130, 3135, and 3140 steels as shown plotted in Fig. 13. For recomputing, however, an average of the values was used.

The number of chips removed per minute is the rpm times the number of teeth in the cutter and equals 46 × 12, or 552, in this case. The feed in inches per tooth is found by dividing the feed in inches per minute by the number of chips removed per minute. The horsepower is equal to the energy in foot-pounds per chip times the number of chips removed per minute divided by 33,000. In this case, E (the energy per chip) = 60 times hp.

Table 7 shows the feed in inches per minute and inches per tooth for 12 different feeds, and the horsepower and energy values for 4 depths and 3 widths of cut and for all feeds for this combination of cutter and material.

Discussion of Ohio State Data. Data for the shell-end mill cutting S.A.E. 3125–3140 steels, as taken from Table 7, are shown plotted in Fig. 14. Three widths of cut, $^1/_8$, $^3/_{16}$, and $^1/_4$ in., were run, and the data are grouped accordingly into three groups. Four depths of cut of 2, 3, 4, and 5 in. were taken for each of 12 feeds, ranging from $^1/_2$ to 20 in. per min. These feeds correspond to 0.00091 to 0.0363 in. per tooth. In Fig. 14 the three sets of curves at the left, indicated by F, show energy values in foot-pounds plotted over feed per tooth as abscissas. In the upper two cuts, for the $^3/_{16}$ and $^1/_4$ in. widths, the points are seen to lie very consistently on straight lines. In the lower set for the $^1/_8$ in. width, however, the points appear to lie on a line slightly concave for the lighter feeds. The solid lines in this set at the right have the same slope as those of the two upper groups, while the dotted lines represent the energy data for these light feeds.

The average slope of the solid lines is 0.833, while that for the dotted lines is 0.68. The slopes of the 4- and 5-in.-depth curves of the $^3/_{16}$-in.-wide cuts and the 4-in.-depth curves of the $^1/_4$-in. cuts are slightly lower than the average. For these lines, it will be noticed that the energy values are relatively higher for the lighter feeds. This may indicate the influence of a dull cutter. The slope of the 2-in.-depth curve of the $^1/_4$-in.-width cuts is higher and the energy values are also somewhat lower, compared with the other data, toward the lighter feed end of the curve, which may indicate that the cutter was a little sharper than average.

It is known that the energy per chip varies very nearly as the first power of the width of cut, other conditions remaining the same. However, the relatively high values of the points on the dotted lines probably are not due to dulling, as they are too consistent, but may be due to the radius of the cutter teeth nose, which would have much more effect on this narrow cut, especially at smaller feeds.

The data for the two widths of $^1/_8$ and $^3/_{16}$ in. have been replotted over the various depths of cut, resulting in the two sets of curves at the right in Fig. 14, labeled *D*. Apparently, a change in the slope of the variable-feed lines for the $^1/_8$-in.-width cuts has had little effect on the variable-depth curves. The average slope for both sets is 0.903.

From the slopes of the variable-feed and variable-depth curves, a general milling-energy formula for this material and cutter can be written as

$$E = 14{,}600\, wf^{0.83}d^{0.90}$$

in which E is in foot-pounds per chip, f the feed per tooth in inches, d the depth of cut in inches, and w the width of cut in inches, as illustrated in Figs. 12 and 13.

From Fig. 14 several interesting and valuable facts are noted. A formula of the type $E = Cwf^xd^y$ is found to hold, even when the depth of cut equals the diameter of the cutter. As is mentioned farther on under "Mathematical Analysis," for this latter condition there is no difference in cutting action between up and down milling. The curves verify the fact that the dulling of a cutter not only increases the energy values, but reduces the slope of the curves. The reduced slope and high values indicated by the curves for the smaller feeds of the narrow $^1/_8$-in.-wide cuts may involve the effect of nose radius.

Fig. 15 shows graphically on log-log paper the energy data for an alternate tooth slotting cutter 6 in. in diameter and 1 in. wide. It has 24 teeth, having 10-deg rake and 15-deg alternate helix. S.A.E. 3125–3140 steels were used, and six depths, namely, $^1/_8$, $^1/_4$, $^3/_8$, $^1/_2$, 1, and $1^1/_2$ in. were run, each with 12 feeds varying from 0.000488 to 0.0195 in. per tooth. Two sets of curves are shown. Those labeled *F* at the left are for variable feed and

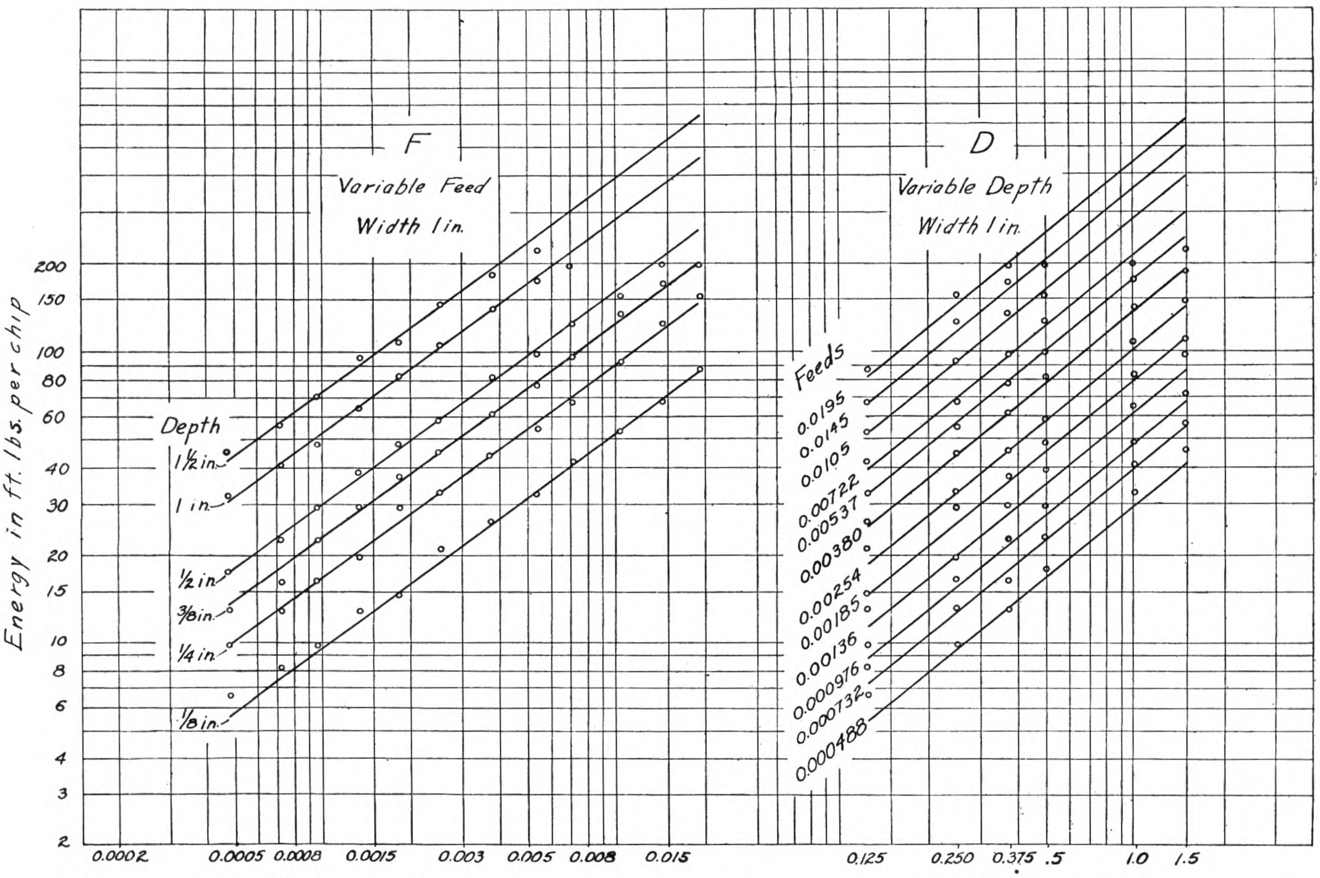

FIG. 15 VALUES OF ENERGY AS COMPUTED FROM THE OHIO STATE EXPERIMENTS FOR ALTERNATE-TOOTH SLOTTING CUTTERS 6 IN. IN DIAMETER, 1 IN. WIDE, HAVING 24 TEETH, EACH HAVING A RAKE ANGLE OF 10 DEG AND A SPIRAL ANGLE OF 15 DEG

(Cutting S.A.E. 3125–3140 steels and plotted on log-log paper for various combinations of feed, depth, and width of cut. An emulsion of 1 part soluble oil to 20 parts water was used. The formula developed from these data is $E = 8800\, wf^{0.735}d^{0.83}$.)

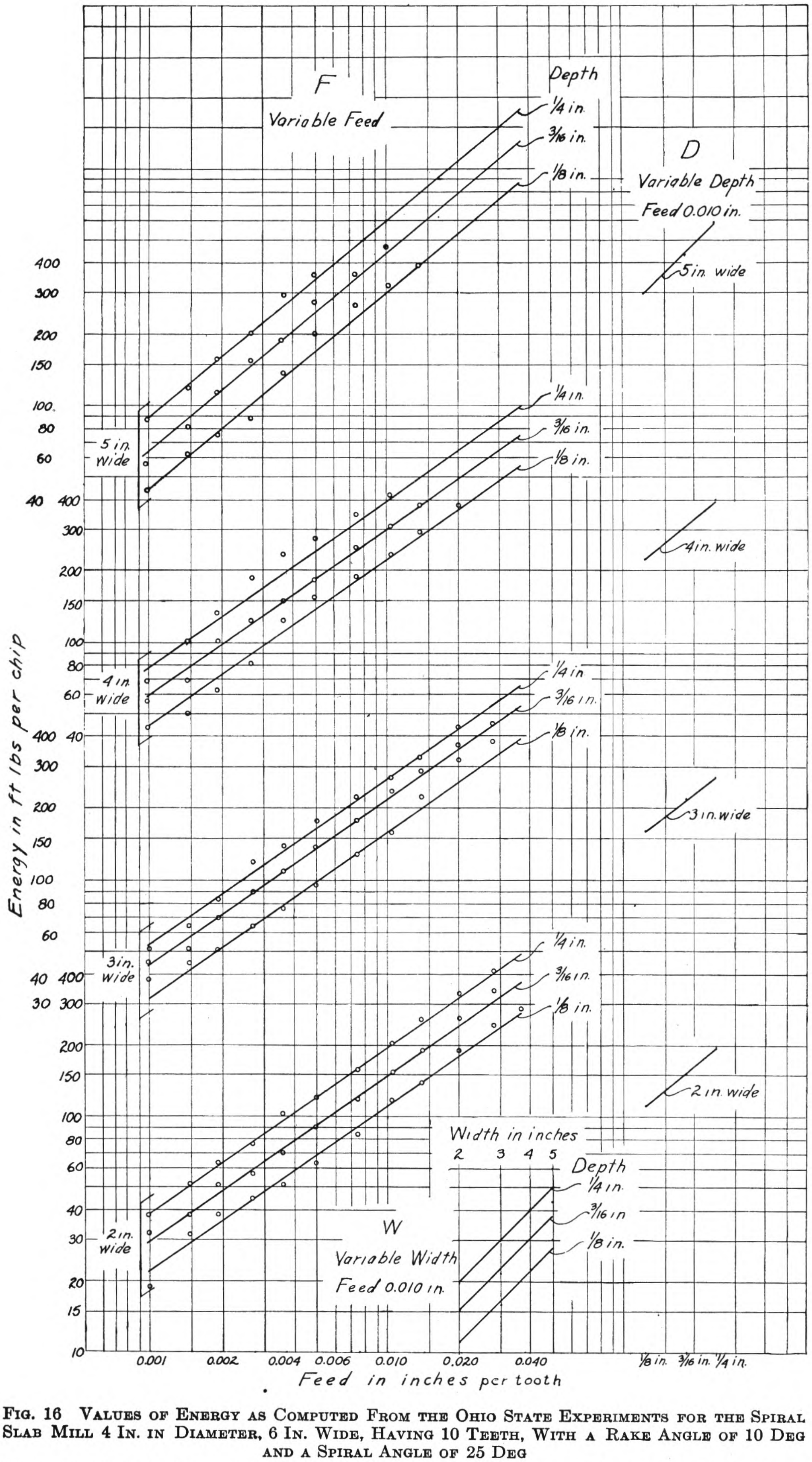

Fig. 16 Values of Energy as Computed From the Ohio State Experiments for the Spiral Slab Mill 4 In. in Diameter, 6 In. Wide, Having 10 Teeth, With a Rake Angle of 10 Deg and a Spiral Angle of 25 Deg

(Cutting S.A.E. 3125–3140 steels and plotted on log-log paper for various combinations of feed, depth, and width of cut. The 1-to-20 emulsion was used. The general formula obtained is $E = 8000\ wf^{0.72}d^{0.81}$.)

constant depth, with the energy values plotted over the feed, and those labeled D at the right are for variable depth and constant feed, with the energy values plotted over the values of depth.

The data are exceptionally consistent, and, due to the comparatively large number of depths run, the slope of the variable-depth lines is well defined. The slopes of the two sets have been measured and result in the equation for energy values per chip for this cutter and material, as follows:

$$E = 8800\ wf^{0.74}d^{0.83}$$

It was assumed that the length of each tooth was equal to the width of the slot, inasmuch as definite information was lacking. This is not correct, however, so the constant given is slightly lower than is actually true.

The energy values for a spiral slab mill when milling the S.A.E. 3125–3140 steels are shown plotted in Fig. 16. The cutter was 4 in. in diameter, 6 in. wide, and had 10 teeth, with 10-deg rake and 25-deg helix. Four widths of cut of 2, 3, 4, and 5 in. were run for each of three depths of 1/8, 3/16, and 1/4 in. The 12 feeds used ranged from 0.000953 to 0.0381 in. per tooth. Four sets of variable-feed curves, labeled F at the left in Fig. 16, result from the recomputation. Of these, three sets, namely, the curves for widths of 2, 3, and 4 in., have approximately the same slope of 0.72, while the set for the 5 in. width is somewhat steeper with a slope of 0.81. Variable-depth curves are not shown because of insufficient number of depths run and general inconsistency of the data. To estimate what this exponent would be, points have been taken from the straight lines and plotted, resulting in curves D. The average slope of these lines is 0.81, which probably is not exact.

These same points have been plotted over variable-width values in an attempt to check the assumption that energy values vary directly as width. These curves are shown at W, but the data are too inconsistent to draw definite conclusions. The resultant straight lines shown have a slope of 1.0, which would be expected for this range of widths.

The milling-energy formula for this cutter and material is then

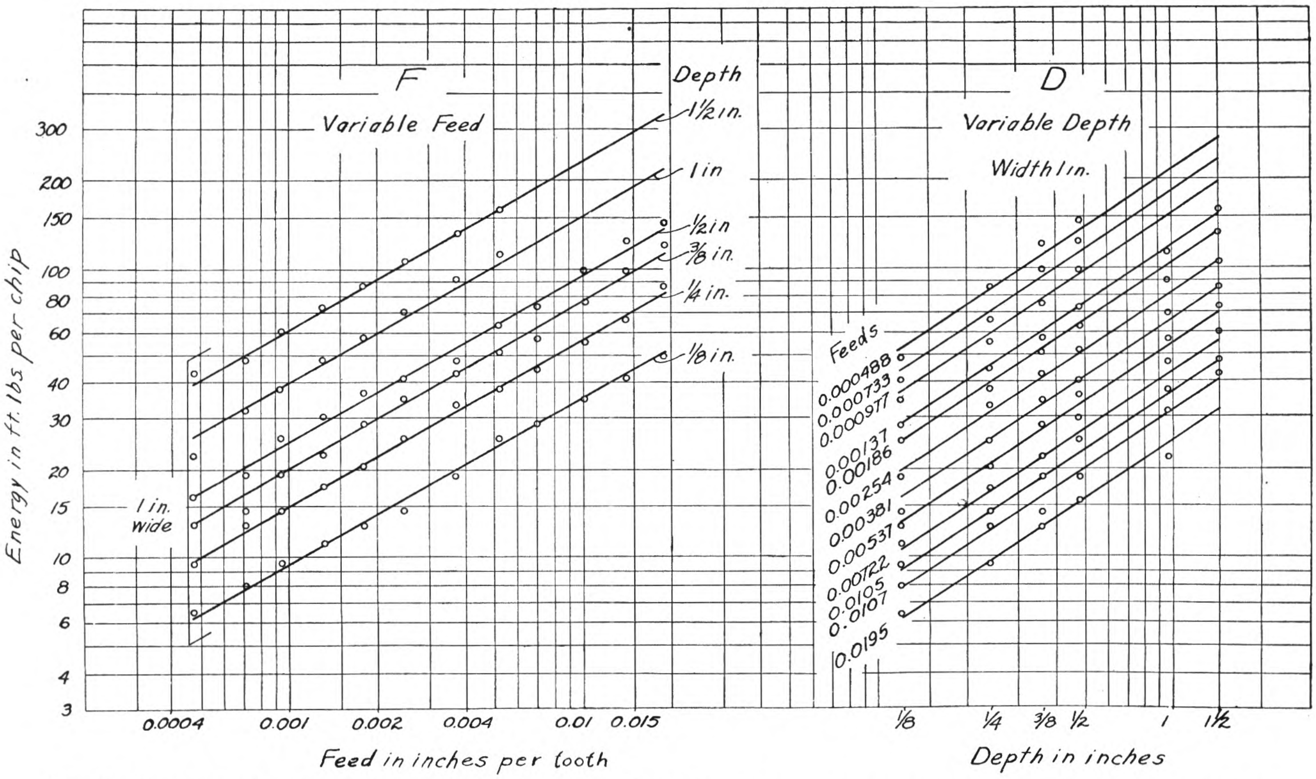

Fig. 17 Values of Energy as Computed From the Ohio State Experiments for the Alternate-Tooth Slotting Cutter Described in Fig. 15

(Cutting cast iron dry, as summarized in Table 8. The energy equation is $E = 2000\ wf^{0.56}d^{0.66}$.)

$$E = Cwf^{0.72}d^{0.81}$$

in which $C = 8000$.

Recomputed values of the Ohio State Experiments are given for the alternate-tooth slotting cutter cutting cast iron dry. The values are shown graphically in Fig. 17, with the energy values for a constant width of 1 in., for various depths from $^1/_8$ to $1^1/_2$ in., plotted over the feed in inches per tooth. A series of parallel lines are obtained, the slopes of which all equal 0.56. When the energy values for constant feed are plotted over depth of cut in inches, again a series of practically parallel straight lines are obtained the slopes of which are 0.66. This gives an energy equation as developed from these series of lines of

$$E = 2000\ wf^{0.56}d^{0.66}$$

Recomputed values of energy in foot-pounds per chip for the half-side mill cutting cast iron are shown plotted over the feed in inches per tooth in Fig. 18. Four sets of curves are shown, for widths of cut of $^1/_8$, $^3/_{16}$, $^1/_4$, and $^3/_8$ in. These data are seen to be more erratic than any other presented. For each width of cut, four lines indicate the general tendencies of energy fluctuations over feed per tooth, for four depths of cut of $^1/_2$, 1, $1^1/_2$, and 2 in. While there are numerous inconsistencies, the general tendency of the lines is to slope from the horizontal at an angle whose tangent is 0.71. The results were quite inconsistent for accurate determination of the slope of the variable depth lines for constant feed, although a slope of 0.91 had been obtained as representing an average. This gives the general energy equation in foot-pounds per chip as

$$E = 5300\ wf^{0.71}d^{0.91}$$

It is believed that, in the case of half-side milling in which only one cutter is mounted on the arbor, an end thrust is imposed on the spindle bearing which gives rise to frictional forces which materially influence the power developed by the motor. This influence is not accounted for in the calibration.

Graphs similar to those shown in Figs. 14 to 18, inclusive, were prepared for the half-side mill cutting S.A.E. 3125–3140 steels, and for the shell-end mill, slabbing mill, and a face mill when cutting cast iron dry. For brevity, however, only Figs. 14 to 18 are presented here. Data for all of these curves are summarized in Table 8.

In milling the S.A.E. 3125–3140 steels with the 1-to-20 emulsion with the shell-end mill, alternate-tooth slotting mill, slabbing mill, and half-side mill, various formulas and constants were obtained as summarized in Table 8. It is seen, for instance, that the energy in foot-pounds per chip formulas for slotting and slabbing are practically identical with the coefficients of feed and depth and also the constants of the slotting mill only slightly higher than those for slabbing. The rake angle of the two cutters is 10 deg, while the helix angle is 15 deg for the slotting mill and 25 deg for the slabbing mill. The results confirm those of the authors' experiments that a difference in helix angle causes very slight, if any, differences in the equations. The energy equation for the half-side mill cutting steel has exponents only slightly higher than the corresponding ones in the slotting and slabbing mill equations. It might be expected that this equation would be between those for slotting and slabbing, inasmuch as it represents only one-half side of the slotting. This is not borne out, however, and, the constant itself being considerably higher than the constants for the slotting and slabbing mills, would indicate greater friction on the spindle bearings due to end thrust. There appears to be considerable difference between the equations for the shell-end mill and the half-side mill. This, from analysis, would not be expected, inasmuch as they have the same rake and helix angles. The constant for the

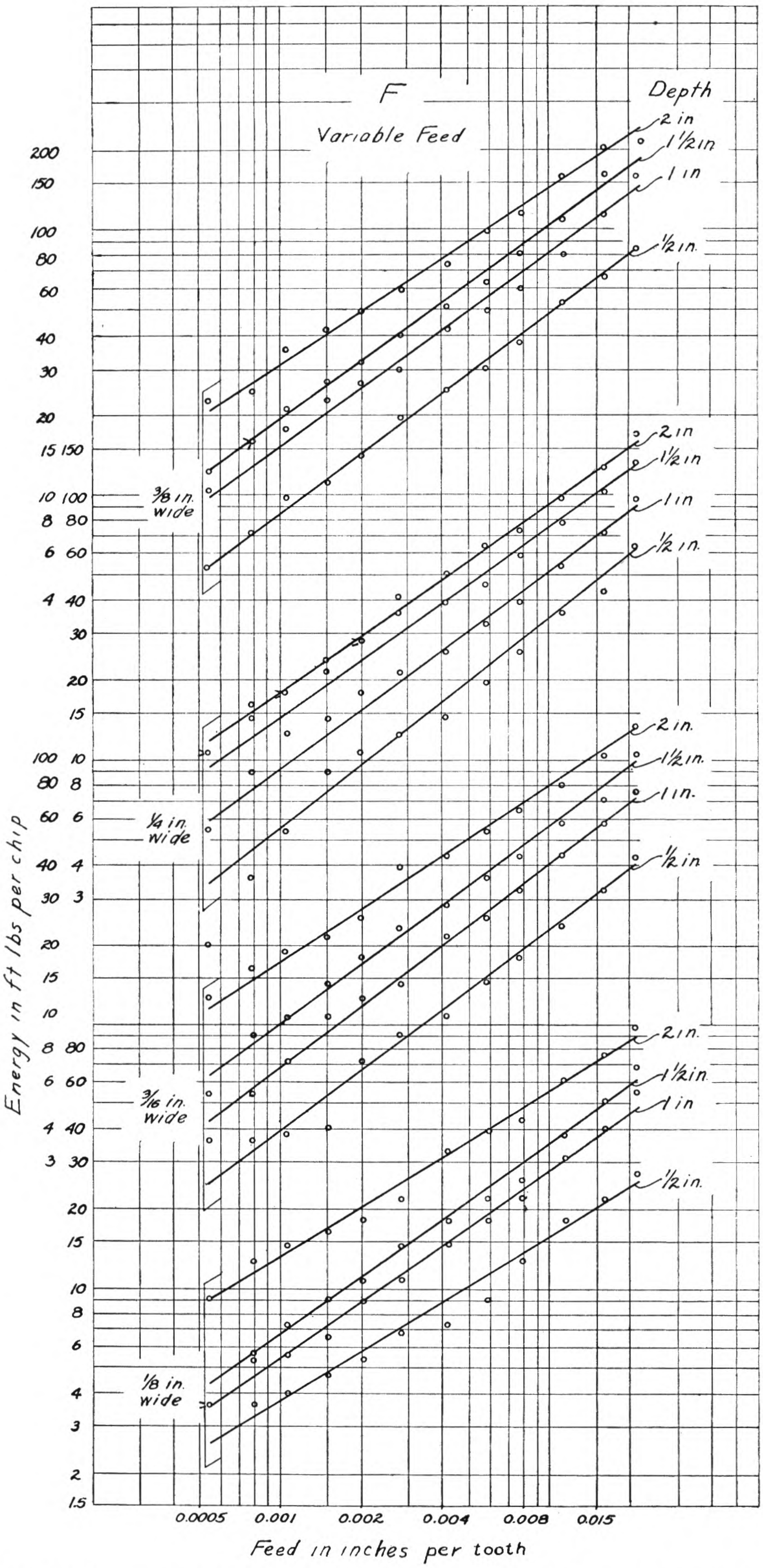

Fig. 18 Values of Energy as Computed From the Ohio State Experiments for the Half-Side Mill 6-In. in Diameter, Having 24 Teeth With 10-Deg Rake and 15-Deg Helix

(Cutting cast iron dry, under the conditions summarized in Table 8. The general equation developed is $E = 5300wf^{0.71}d^{0.91}$.)

shell-end mill formula is greatest of all cutters, due presumably to greater end thrust on the spindle, inasmuch as the depth of cut ranges from 2 to 5 in., whereas that for the half-side mill is from $^1/_2$ to 2 in.

At the extreme right in Table 8 are shown values of horsepower per cubic inch per minute for each of the various cutters when taking 0.010 in. feed per tooth and a depth of 1 in. The slabbing mill is most efficient, while the half-side mill is least efficient.

An analysis of the equations resulting from the use of different cutters when machining cast iron, as given at the bottom of Table 8, shows the effect of the same tendencies as when milling steel, as previously discussed. The horsepower per cubic inch per minute is least when using the slotting and slabbing cutters and greatest when using the face mill. That for the face mill is slightly greater than that for the side mill, due again presumably to the additional thrust because of the great depth of cut, 4 to 8 in.

From the data shown in Table 8, it is obvious that a half-side type of cut does not give values midway between that for the slabbing mill and the slotting mill, presumably due to the influence of friction on the spindle due to end thrust. This might be true where the half-side cuts are made by straddle milling, in which no thrust is put on the spindle.

A Mathematical Analysis of Cutting Formulas

In concluding this paper, the authors wish to present a mathematical analysis of milling-energy formulas. These formulas have all been of the form

$$E = Cwf^x d^y$$

where, in most cases, the value of the constant C was different for up and down milling, and often the values of the exponents x and y also were different.

When d, the depth of cut in inches, equals one-half the cutter diameter, the greatest difference between up and down cutting is obtained, but as it increases beyond one-half of the diameter, the two methods of cutting approach each other. When d is equal to the cutter diameter, the two methods are identical, and it is obvious that, for this case, the energy values E for a specific chip size will be the same for up and down milling. This condition occurs commonly in face-milling operations where the width of the cut, corresponding to the depth of cut in these investigations, is often the full width or diameter of the cutter.

Let subscript u denote up milling and subscript d denote down milling. Also, let d, the depth of cut in inches, equal D, the cutter diameter in inches, which is a constant. For this condition, then

$$E_u = C_u wf^{x_u} D^{y_u}$$

$$E_d = C_d wf^{x_d} D^{y_d}$$

TABLE 8

(A summary of the results of the Ohio State Experiments, together with the recomputed data based on feed and depth of cut)

Material and cutting fluid	Cutter: Kind	Diam and width	No. of teeth	Tool angles: Rake, deg	Tool angles: Helix, deg	Speed: Rpm	Speed: Min	Range of test data in inches: Width	Feed	Depth	Energy per chip formulas	Value of constants	Horsepower per cu in. per min formulas	Horsepower per cu in. per min[a]
S.A.E. 3125–3140 steels; 20-to-1 Sunoco emulsion	Shell-end mill	5″ d	12	10	15	46	60	1/8 to 1/4	0.00091 to 0.0363	2–5	$Cwf^{0.83}d^{0.90}$	14600	$\frac{C}{33000\,f^{0.17}d^{0.10}}$	0.99
	Slotting[b] mill, alternate tooth	6″ d 1″ w	24	10	15	43	68	1	0.000488 to 0.0195	1/8 to 1 1/2	$Cwf^{0.74}d^{0.83}$	8800	$\frac{C}{33000\,f^{0.26}d^{0.17}}$	0.885
	Slabbing mill	4″ d 6″ w	10	10	25	53	55	2–5	0.000953 to 0.0381	1/8 to 1/4	$Cwf^{0.72}d^{0.81}$	8000	$\frac{C}{33000\,f^{0.28}d^{0.19}}$	0.877
	Half-side mill	6″ d	24	10	15	38	60	1/8 to 1/8	0.00055 to 0.0218	1/2 to 2	$Cwf^{0.75}d^{0.85}$	10700	$\frac{C}{33000\,f^{0.25}d^{0.15}}$	1.05
Cast iron, dry	Shell-end mill[c]	5″ d	12	10	15	46	60	1/8 to 3/8	0.00091 to 0.0364	3–5	$Cwf^{0.60}d^{0.90}$	3100	$\frac{C}{33000\,f^{0.40}d^{0.10}}$	0.592
	Slotting[d] mill, alternate tooth	6″ d 1″ w	24	10	15	43	68	1	0.000488 to 0.0195	1/8 to 1 1/2	$Cwf^{0.56}d^{0.66}$	2000	$\frac{C}{33000\,f^{0.44}d^{0.34}}$	0.460
	Slabbing mill	4″ d 6″ w	10	10	25	62	65	2–6	0.000807 to 0.0322	1/8 to 3/8	$Cwf^{0.51}d^{0.67}$	1600	$\frac{C}{33000\,f^{0.49}d^{0.33}}$	0.465
	Half-side mill	6″	24	10	15	38	60	1/8 to 3/8	0.00055 to 0.0218	1/2 to 2	$Cwf^{0.71}d^{0.91}$	5300	$\frac{C}{33000\,f^{0.29}d^{0.09}}$	0.610
	Face mill (inserted blade)	8″ d	14	15	7	29	60	1/8 to 1/4	0.00125 to 0.050	4–8	$Cwf^{0.52}d^{0.90}$	2300	$\frac{C}{33000\,f^{0.48}d^{0.10}}$	0.635

[a] 0.010-in. feed, 1.00-in. depth. [b] Two other slotting mills, 1/4 and 1/2-in. wide, were tested. [c] A 2-in.-diameter shell-end mill was tested also; the data were inconsistent. [d] A 1/2-in.-wide slotting cutter also was tested.

But, $E_u = E_d$. Then, by transposing and eliminating w, the equations become

$$\frac{C_u\,D^{y_u}}{C_d\,D^{y_d}}f^{x_u} = f^{x_d}$$

For convenience in this analysis, let

$$\frac{C_u D^{y_u}}{C_d D^{y_d}} = K$$

Then

$$Kf^{x_u} = f^{x_d}$$

By taking logarithms,

$$\log K + x_u \log f = x_d \log f$$

or

$$x_d - x_u = \frac{\log K}{\log f}$$

Since f is always less than 1, $\log f$ is always negative.

This becomes a very illuminating equation, showing that the difference between the feed exponents for up and down milling is a function of the log of K. The farther the value of K is from 1, the greater will be the difference between these exponents. If K equals 1, then $\log K$ equals zero, and the two exponents must be equal. For this case

$$C_u D^{y_u} = C_d D^{y_d}$$

If the values of C are not the same, which in general is true, then the depth exponents y_u and y_d cannot be equal.

Similarly, when K does not equal 1, x_u is greater than x_d if $C_u D^{y_u}$ is greater than $C_d D^{y_d}$. And conversely, x_u is less than x_d, if $C_u D^{y_u}$ is less than $C_d D^{y_d}$.

The equation also points out that the difference between the feed exponents in the up and down milling equations is not constant, but is a function of the log of the feed. If the difference varies, the exponents also must vary and the variable feed lines cannot be absolutely straight lines on log-log paper. This effect will be very small when K is nearly unity.

Examination of the experimental data will reveal many cases which do not check this theoretical analysis. The reason is obvious from the foregoing analysis. An equation of the form $E = Cwf^x d^y$, while accurate enough for all practical purposes, is not strictly correct. The exponents themselves are a function of the variables f and d involved.

Conclusions

There are numerous final conclusions drawn from the single-point cutter data and the Ohio State Experimental data which may be summarized briefly as follows:

Single-Point Cutter (when milling the free-cutting brass):

1 All exponents of feed and depth and constants in the energy equations are lower for land than for groove cutting, due presumably to greater friction in groove cutting or because the chip is cut at each side, as well as along the periphery when groove cutting, as shown in Fig. 2.

2 The exponents of the feed and depth are found to be the same for up and down milling for groove cutting and also different but the same for land cutting.

3 Constants in the energy equation are lower for down milling for the same type of cut.

4 Increasing the rake angle materially decreases the energy value E, as shown in Fig. 3.

5 Increasing the helix angle has no consistent influence on the exponents, constants, or energy until high values are obtained which produce excessive friction.

(When milling S.A.E. 1020 steel):

6 The conclusions given for free-cutting brass hold in general.

7 It is found that the exponents of feed and depth are slightly different when cutting up than when cutting down, as indicated in Fig. 4.

8 The drop in energy due to higher values of rake angle is very appreciable, as indicated in Fig. 5. This falling off of energy with increased rake angle appears to become much less rapid when the rake angle is 25 deg. The value of E for the 0-deg rake angle is over 100 per cent more than that for the 25-deg rake.

(When milling S.A.E. 1112 steel):

9 The results obtained, as shown in Figs. 6 and 7, confirm those for the S.A.E. 1020 steel. The percentage drop in energy due to rake angle is less pronounced.

(*When milling S.A.E. 3250 steel*):

10 There appears to be little or no difference in values of exponents, constants, or energy values when milling up or down. For this steel it appears that the values of the feed and depth exponents are somewhat higher for land cutting than for groove cutting, as indicated in Fig. 8.

11 The energy values decrease uniformly with increase in rake angles up to 20 deg, but show that the energy for groove milling is greater than for land milling shown in Fig. 9.

(*By comparing the results obtained from the three steels discussed*):

12 It is seen that the S.A.E. 1112 steel gives energy values slightly below the average for the three steels, the S.A.E. 1020 steel gives values about equal to the average of the three, while the S.A.E. 3250 gives values slightly above the average steel.

13 Steels S.A.E. 1020 and S.A.E. 1112 give lower values of energy when cutting down than when cutting up, while S.A.E. 3250 shows very little difference in energy values for these two types of cuts.

14 The constants for cutting down for all steels averaged as shown in Fig. 10 are considerably lower than when cutting up in all cases except that for the 0-deg rake. The average constant for the 25-deg rake angle is about 55 per cent of that for the 0-deg rake angle. There is a tendency for the constant curves to become horizontal for the greater values of rake.

15 The constants for land cutting are considerably lower than the corresponding values for groove cutting.

16 The constants, when plotted over the side rake or helix angles in Fig. 10, are seen to remain practically constant.

17 The average horsepower per cubic inch per minute values for all steels are shown in Fig. 10 to fall off uniformly with an increase in rake angle until a 20-deg rake angle is reached, after which there is little additional reduction in power values for an increase in rake angle.

18 The power values remain practically constant for various values of helix angle when the rake angle is constant. For groove cutting, the energy increases perceptibly for helix angles above 20 deg, due presumably to increased friction.

19 Energy values for groove cutting are shown to be higher than those for land cutting, due presumably to the types of cut.

20 It is shown in Fig. 11 that the values of the exponents of feed and depth remain practically constant for all values of rake and helix. Variations are introduced for high values of helix angle in groove cutting, however.

The Ohio State Data:

21 After reworking all of the Ohio State experimental data and plotting energy values on a basis of feed per tooth and depth of cut as variables, the conclusion is reached that energy equations of the form $E = Cwf^x d^y$ hold for all combinations of feed and depth, even for values of depth of cut up and equal to the diameter of the cutter.

22 It also is shown that this formula holds alike for plain mills, half-side mills, side mills when groove cutting, shell-end mills, and face mills. It must be remembered, however, that the depth of cut in these cases is measured in the plane of the diameter of the cutter, while the width is measured axially, as shown in Fig. 12. Half-side cuts, as shown at *C, D,* and *E* in Fig. 12, do not give equation values midway between slabbing cuts as shown at *A* and groove cutting as shown at *B*, as might be expected from an analysis. This is due, undoubtedly, to the unbalanced effect of a single cutter. This condition would, it is believed, be rectified if half-side cuts were taken with two opposed cutters so as to eliminate the end thrust on the spindle.

General Conclusions:

23 It has been found from all experiments with the single-point cutter, as well as from the commercial type of cutter of various classes, that all milling-energy data can be expressed by the equation $E = Cwf^x d^y$.

24 It is shown that the net power values of commercial milling, when reduced to energy in foot-pounds per chip as a function of feed and depth of cut, agree very closely with corresponding values obtained from the single-tooth cutter in the pendulum-type machine. It is believed that for research purposes, the values obtained with the pendulum-type machine are more accurate than those obtained with the commercial machine, particularly where net motor horsepower, as measured by a wattmeter, is used.

25 It is shown that the different materials respond differently to the same change in front-rake angle of the tool, as far as energy is concerned. One metal may be removed much more efficiently with a high rake angle than with a low rake angle, while another may be removed with only slightly greater efficiency. This also has been shown previously in Fig. 26 of a paper, "Research in the Elements of Metal Cutting."[6]

26 Different exponents and constants are obtained in the energy equation for the same tool when cutting different materials. Each material seems to give rise to its own characteristic constants and exponents. Small changes in structure or composition of a metal also will produce these changes.

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Discussion

Hans Ernst.[7] The investigation described in this paper is an excellent extension of the field covered by "Elements of Milling, Part 1." It is gratifying to see that this investigation has covered the effect of helix angle on both land and groove cutting; also the effect of front rake in both cases.

Many interesting points are brought out in the tables and charts that accompany the paper. Chief among these is the very marked decrease in the energy per chip as the rake angle is increased. This is particularly true in the case of S.A.E. 1020, the large reduction in this case being probably due to the ductile nature of this material.

A peculiar feature of the test on free-cutting brass is the considerable increase in energy value per chip between 0- and 10-deg helix angle when cutting on a land. It is difficult to account for this, as one would normally expect a gradual reduction between 0 and 10 deg, just as has been found from 10 to 40 deg. The increase in energy with increase in helix angle when milling in a groove is of course readily explained by the increased opposition to free flow of the chip by interference with the size of the groove.

It is hard to account for the slight rise in the energy value per chip which was found in some cases when the rake angle

[6] Trans. A.S.M.E., vol. 48, 1926, pp. 749–848.

[7] Research Engineer, Cincinnati Milling Co., Cincinnati, Ohio. Mem. A.S.M.E.

(when cutting on a land) was increased from 20 deg to 25 deg. In the case of S.A.E. 3250, one might reason that with this tough material the vertical component of the chip pressure against the nose of the cutting tool was sufficiently great to deflect it downward against the work surface, thus possibly causing an additional drag thereon, but this argument appears to fall to the ground in the case of the free-cutting brass where a similar increase in power is noted, yet where the chip pressure must be very much lower.

The somewhat erratic changes in the values of the feed and depth exponents (particularly the latter) is likewise difficult to explain. It is possible that this may be due to chance differences in the smoothness of the lip surface of the cutting teeth, when changing from one rake angle to another. In our own work, we have found that the condition of lip surface has a tremendous influence on the formation of the built-up edge. In fact, I have recently found in the case of a special cemented carbide that no built-up edge whatever was produced under the same conditions that, with other cutting materials, would produce a very substantial built-up edge. Obviously, as the formation of the built-up edge is usually a function of the instantaneous chip thickness, it would seem that very slight differences in the character of the lip surface of the tool would have a marked effect upon the value of the depth exponent.

It is interesting, and important, to note the general correspondence between the results obtained in the Ohio State experiments and those obtained with the single-point tool, though here again the perplexing variations in the values of the feed and depth exponents show how dependent all the energy values are upon the particular mechanism of chip formation existing in the case of each test.

In commenting on the finish produced on up and down milling it is mentioned that, in cutting up, "the chip has a tendency to cling to the cutting edge, and, if left on, would be dragged under the cutter on the next cut, marring the surface." From our experience we believe that such a condition depends largely on the material, feed per tooth, and cutting fluid used and can scarcely be considered as a condition inherent in up cutting. In commercial milling, the finish with down cutting is usually worse than with up cutting on ductile materials such as the low-carbon steels, but may be better in the case of high-carbon steels or cast iron.

In commercial milling we have found that the chips in down milling are usually flatter than in up milling, which is contrary to the results reported in the paper. Here again, the action of the built-up edge on chips produced under commercial conditions may be responsible for the difference.

The authors should be commended for their painstaking work in the conducting of these tests and their analysis of the results obtained.