Problems of Modern Pump and Turbine Design

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This is a general review of the present status of design principles which apply equally to centrifugal pumps and hydraulic turbines. The author first develops step by step the fundamental ideas underlying the theoretical behavior of a runner having an infinitely large number of thin blades which guide the fluid perfectly. The resemblance between the blading ordinarily used and the wing of an airplane has led to analysis on the basis that each blade acts as an airfoil. The author points out the many limitations of this theory and advocates a new beginning, taking as a starting point the theoretically perfect runner. By decreasing the number of blades first from infinity to a finite but very large number, he analyzes the forces and influences that are at work. Although he does not claim to have arrived at a final solution, he demonstrates the play of one effect upon another, and suggests that further experimentation should assist in making proper allowances for the uncertainties not subject to strict analysis and which become more and more important as the number of blades is reduced. The final section deals with the best conditions for the design of high-speed runners. The influence of the principal figures of layout upon which both efficiency and cavitation are dependent are discussed, and complete curves for efficiency and cavitation coefficient are given for three different specific speeds. This speed is defined by the same figure for pumps and turbines. He feels that both theories in spite of their defects can be used to advantage as experimental knowledge is progressively increased, and demonstrates that mathematics will throw light on a number of vital problems dealing particularly with efficiency and cavitation. He suggests for consideration a special type of profile, and expresses the hope that by combining the laboratory with theory, it will be possible to realize improvements in existing equipment, and finally to build up a strict method for pump and turbine design meeting properly and satisfactorily all of the requirements for high specific speed and small number of blades.

THE design of turbines and pumps has been developed extensively during the last 15 years. Possibly the progress in turbine design has had more recognition because the problems connected with the building of hydraulic turbines are more commonly appreciated. Pumps are usually of comparatively small capacity, and serve purposes which do not appeal so readily to the popular imagination, whereas the conditions for which turbines are designed are more exacting and vary widely from one installation to another. In many cases special effort must be taken to fulfil the requirements of the particular installation, in order to adapt the design to the given specifications for head and discharge, together with the desire of the customer regarding speed. On the other hand, pumps are often manufactured for stock, as a design once developed may be readily adapted to meet special requirements.

It should be stated that to the present time both branches have progressed hand in hand, and are together benefiting by the achievements of modern hydrodynamics. Although modern theoretical and experimental methods, or rather the ideas upon which these methods are based, are successfully used in the design of both high- and low-head machinery, it is in the low-head design that these are most important. Furthermore, it is true that practical ability backed by engineering judgment must still take the place of strict mathematical rules and formulas.

For this reason there are as yet many unsolved problems, but in the field of turbines in general these appear to be most critical for low-head machines operating at high specific speeds. It is the purpose of this paper to discuss these questions without claiming to give exact and final solutions, but attempting to point out the essence of the problems, and to show in which directions the answers may possibly be found.

In order to have a general background for this discussion, an understanding of the turbine principle is necessary.

1 The Turbine Principle—Fundamental Equations

Fig. 1 shows a sectional elevation of a vertical runner which could be used either as a turbine or as a pump—i.e., to transform energy of flowing water to mechanical energy or vice versa. The efficiency as one or the other depends primarily upon the shape of the blades, but in principle it is possible to use the same blading for either. It will be seen that there are guide vanes above and outside the runner, whereas in the throat below the runner no means of guiding the water is used.

The arrangement shown is quite usual for turbines, and for pumps it is used frequently. As far as an analysis of conditions is concerned, it makes no difference whether the flow, approaching or leaving, passes through a scroll case or an open flume, or any other type of passage. Fig. 1 shows, therefore, only the guide apparatus and the runner.

According to the theory of conservation of momentum, a torque will be produced in the shaft providing the runner changes the "moment of whirl" of the fluid. One of the fundamental principles of mechanics is that force of impact or reaction in pounds is equal to weight per second, which is equal to the dis-

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charge Q in cubic feet per second, times the weight per cubic foot γ, times the change in velocity, divided by the acceleration of gravity. It follows, therefore, that if the tangential component of velocity v_{tang} is substituted for velocity, and force may be multiplied by the radius to give torque in pounds perpendicular feet, this fundamental law may be expressed by the following equation:

$$T = \frac{W}{g}\left\{(v_{tang}\, r)_2 - (v_{tang}\, r)_1\right\} \qquad [1]$$

The torque is therefore proportional to the amount of water flowing and to the change produced by the blading in the moment of the whirl, the subscripts $_2$ and $_1$ indicating the conditions after

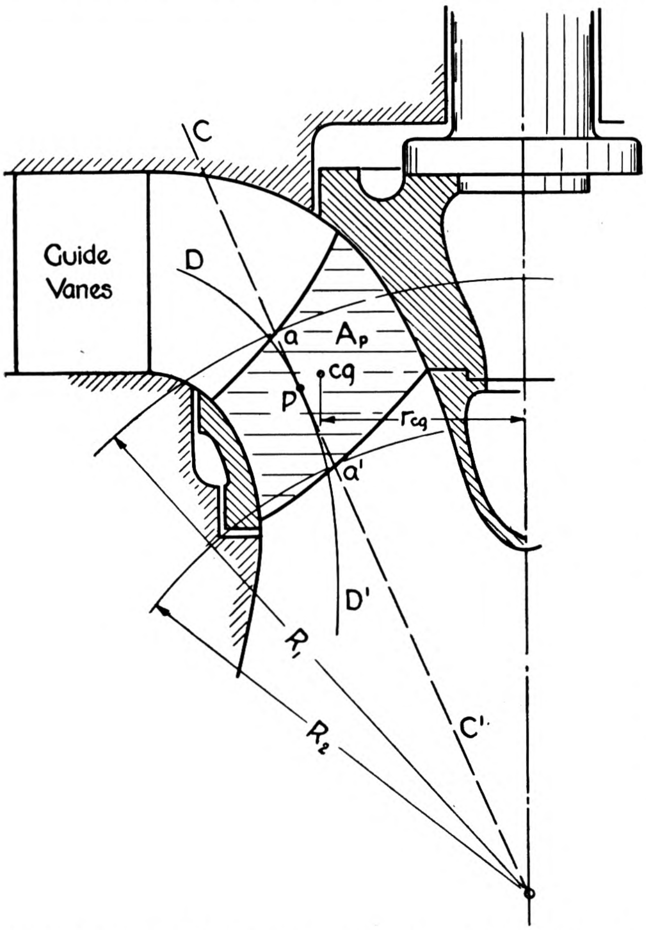


FIG. 1 SECTIONAL ELEVATION OF A HIGH-SPEED PUMP OR TURBINE

and before passing the runner, respectively. From this it follows that it is essential for a pump or turbine to produce in the whirl an alteration, which is termed the "deflecting effect."

It will be appreciated that to have torque it is not necessary that the runner should revolve, as a runner blocked in a stationary position would exert a twist in the shaft even though it acted on the water passing through it simply as a set of guide vanes. When the shaft does rotate, then mechanical work either has to be furnished from some outside source, such as a motor, or else has to be absorbed in driving some other machine, such as an electrical generator.

In order to be consistent in the use of signs, torque will be considered positive when the outside force acts in the direction that the runner is turning, since in the case of a pump the moment of whirl at exit then exceeds that at entrance, giving the right-hand term of the foregoing formula a positive value. On the other hand, the torque for a turbine will be considered negative as the moment of whirl is reduced by the delivery of energy to the outside.

It should be mentioned in regard to the use of symbols that in order to avoid duplication it may be necessary to depart somewhat from the accepted standard both in America and abroad: v will be used for absolute velocity, thus avoiding the c, which is regularly used abroad, because to the American mind this letter is generally reserved to signify a constant. In order to make the formulas more graphic, the components are denoted by the subscripts $_{tang}$ for tangential, $_{ax}$ for axial, $_{rad}$ for radial, and $_{peri}$ for peripheral, etc., rather than to use arbitrary subscripts which are not easily remembered by the reader. Subscript $_1$ will denote the conditions at the beginning or entrance to the runner, $_2$ at the end or exit, and $_0$ the conditions for approaching flow in the case of a stationary blade.

The next step is to multiply both sides of Equation [1] by angular velocity ω and derive as equation for the power HP:

$$HP = \frac{T\omega}{550} = \frac{W\omega}{550g}\left\{(v_{tang}\, r)_2 - (v_{tang}\, r)_1\right\} \qquad [2]$$

In exchange for the work done per second, there must be in the case of a pump an increase in specific energy of the flow, but a corresponding decrease in the case of a turbine. The specific energy may be measured in terms of a manometer column of height H according to the definition $H = p/\gamma + h + v^2/2g$, which means that the energy per unit of weight is equal to the sum of the potential and kinetic energy, p being the pressure exerted in pounds per square foot, γ the density, or specific weight in pounds per cubic foot, h is the elevation of the point where the pressure is measured, and $v^2/2g$ the velocity head. If we designate the specific energy of the flow at entrance to the runner by H_1 and that leaving the runner by H_2 and assume for the present no losses, then the power, expressed by Equation [2], is also equal to:

$$HP = \frac{W}{550}(H_2 - H_1) \qquad [3]$$

which simply states the familiar expression that power is equal to the weight of water falling or raised, times the head.

By combining [2] and [3], we get the fundamental equation of the turbine theory for ideal conditions:

$$H_2 - H_1 = \frac{\omega}{g}\left\{(v_{tang}\, r)_2 - (v_{tang}\, r)_1\right\} \qquad [4]$$

Now by taking ω into the parentheses and combining with r_2 and r_1 to give u_2 and u_1, the velocities of the runner at entrance and exit, we have

$$H_2 - H_1 = (v_{tang\,2}\, u_2 - v_{tang\,1}\, u_1)/g \qquad [5]$$

In the ideal turbine or pump, in which no losses occur, H_2 minus H_1 is equal to the total or gross head. Actually, account must be taken of the head losses. To do so makes [5] become:

$$H_2 - H_1 = (v_{tang\,2}\, u_2 - v_{tang\,1}\, u_1)/g - H_f \qquad [6]$$

where H_f represents the losses expressed in terms of head.

Keeping in mind the convention in regard to signs, it follows that in the case of pumps $(H_2 - H_1)$ is positive; that is to say, there is a positive increase in the energy of flow. In the case of turbines it is negative, as there is a decrease of energy which is transformed into mechanical energy delivered to the shaft. In the case of pumps the power supplied must always be greater than the power delivered to the water on account of the losses, whereas in the case of turbines the conditions are reversed. To adopt a shorthand expression for that head which regardless of leakage losses, bearing losses, and disk friction corresponds to the torque absorbed or developed by the shaft and referred to in this paper as the net or effective head, we set $H_2 - H_1 + H_f = \Delta_1{}^2H$. Making this substitution, we arrive at the equation

$$\Delta_1{}^2H = (v_{\text{tang }2}\, u_2 - v_{\text{tang }1}\, u_1)/g \qquad [7]$$

which is fundamental to the design of turbines and pumps.

2 Discussion and Application of the Fundamental Formula

The values of v_{tang} in Equation [7] are to be considered as averages with respect to time and space existing in the free passages of the turbine or pump, before or after, or even between the blades. The initial whirl moment $(v_{\text{tang}}\, r)_1$ in a turbine is created by the discharge through the guide vanes, and is determined by their angles and opening, whereas the final whirl moment $(v_{\text{tang}}\, r)_2$ is governed essentially by the discharge, the angles of the blades, and also the cross-sectional area and speed of the runner at its outlet. Fluctuations in smooth flow are caused by the interference of the runner blades and the guide vanes, but these may be neglected, at least for the present, even though it has been shown that they can create vibration if conditions are proper.(1)[2] These whirl moments vary also from place to place, because it is necessary to have a finite number of blades, and it is just for this reason that there are many difficulties in determining exactly the amount of deflecting effect exerted by any system of blading upon the flow.

In the development of the elementary theory, this difficulty was avoided by assuming the number of blades to be infinitely high and the blades themselves to be infinitesimally thin. Thus the fluid when passing through the system is guided exactly by the blade surfaces, and so the angles of relative discharge with respect to the blades may be taken as equal to the blade angles. In addition, a uniform exchange of energy is assumed for all layers or filaments of flow. This is equivalent to the assumption of a constant value of $(v_{\text{tang}}\, r)$ in all passages where no blades or vanes, or other devices such as the scroll case, influence the flow, an assumption which is only true for ideal conditions.

Now in order to apply formula [7] in adopting a proper design, it is necessary to select from all possible values of whirl moment those which promise to be conducive to best efficiency. In the case of pumps there are, as a rule, no guiding devices such as vanes or scroll cases ahead of the runners, and therefore the initial whirl is generally taken as zero. In other words, the flow is assumed to approach the runner with purely axial or radial motion, or a combination of both.

From this it follows that $\Delta_1{}^2H$ is equal to $\omega/g(v_{\text{tang}}\, r)_2$.

On the other hand, turbines are usually designed to secure an axial discharge from the runner. Hence, in this case, the $(v_{\text{tang}}\, r)_2$ is equal to zero, and $\Delta_1{}^2H$ is equal to $\omega/g\ (v_{\text{tang}}\, r)_1$. The meaning of this is that, for both turbines and pumps as ordinarily designed, the difference in energy between outlet and entrance, which is proportional to the effective head, is dependent upon the value of the moment of whirl in the transitional passage between the runner and the guide apparatus, and also upon the angular velocity of the runner.

A given value of $(v_{\text{tang}}\, r)$ may be made up of a large v_{tang} and a small r, or vice versa, and consequently different designs can be derived for definite specific speeds. To compare these ideas with American practise, let us rewrite formula [7] in the form $\Delta_1{}^2H = uv_{\text{tang}}/g$, u and v_{tang} referring to the conditions at the entrance of the turbine or the outlet of the pump. Introducing now a ratio $m = v_{\text{tang}}/u$, which is a characteristic of the velocity diagram, and by substituting mu for v_{tang}, we then have $\Delta_1{}^2H = mu^2/g$.

Furthermore, in American practise it is common to use the coefficient ϕ, which is the ratio between the peripheral speed of the runner and spouting velocity corresponding to the head, which, neglecting losses for the present, may be expressed $\phi = u/\sqrt{2g\,\Delta_1{}^2H}$.

Still assuming that there are no losses, $u = \phi\sqrt{2g\,\Delta_1{}^2H}$. Squaring, we have $u^2 = \phi^2 2g\,\Delta_1{}^2H$. Transposing, $\Delta_1{}^2H = u^2/2g\phi^2$, which, as shown, is also equal to mu^2/g. By canceling u^2 and g, m becomes equal to $\dfrac{1}{2\phi^2}$ and $\phi = \sqrt{\dfrac{1}{2m}}$. Both m and ϕ are characteristic of the velocity diagram, and their special significance will be shown later.

3 Modern Point of View Regarding the Fundamental Formula

Modern high-speed runners must be designed with but a few blades which are not excessively long, as otherwise the high relative velocities would cause too high skin friction and the efficiency would be impaired. Therefore, it cannot be assumed that blades can be made to guide each filament of flow exactly, or that the values of $v_{\text{tang}}\, r$ can be computed exactly from the discharge, the speed, the cross-section, and the blade angles.

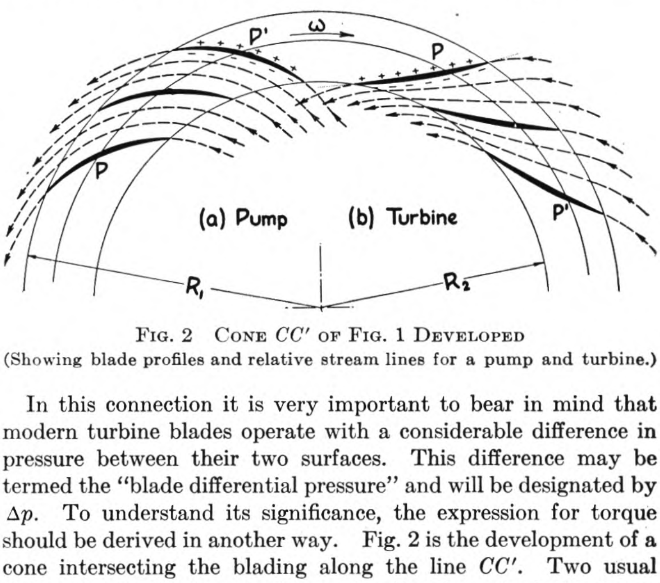


Fig. 2 Cone CC' of Fig. 1 Developed
(Showing blade profiles and relative stream lines for a pump and turbine.)

In this connection it is very important to bear in mind that modern turbine blades operate with a considerable difference in pressure between their two surfaces. This difference may be termed the "blade differential pressure" and will be designated by Δp. To understand its significance, the expression for torque should be derived in another way. Fig. 2 is the development of a cone intersecting the blading along the line CC'. Two usual shapes for the blades are shown, the direction of rotation being the same for both. The circle PP' corresponds to the point P in Fig. 1, located in the horizontally cross-hatched area A, which is a circular projection of the blade against a radial plane. It is evident that the torque of the shaft is equal to the unbalanced force caused by the effect of Δp. As a first approximation let it be assumed that this is uniformly distributed, in which case the torque is equal to the projected area, A_p, in Fig. 1 times the unbalanced pressure Δp, times the radius r_{cg}, again multiplied by the number of blades z. This may be expressed by

$$T = \Delta pA\, r_{cg}\, z = \frac{W}{g}\,(v_{\text{tang }2}\, r_2 - v_{\text{tang }1}\, r_1) \qquad [8]$$

all losses such as leakage, disk, and bearing friction being neglected.

It is possible by the use of this simple expression to compute the average blade pressure, although this is but an average with

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

no way of telling how it is distributed or divided between the push on one surface and the pull on the other. Two facts are obvious from Fig. 2:

(1) As the flow lines approach the leading edge of the blade, they are inclined away from the high-pressure side and bend toward the region of lower pressure. A corresponding effect is present at the trailing edge, and so it follows that the blades must be curved more than would be required to satisfy the average deflection as computed from application of the fundamental formula [7]. This fact, however, does not affect the validity of the elementary theory which assumes an infinite number of blades and perfect guiding of the stream.

(2) Along any concentric circle the pressure varies systematically from one side of one blade to the adjacent side of the next. Referring to the average pressure as computed from the elementary assumptions as zero, the pressure at one end of the distance separating two adjacent blades is minus and plus at the other, in much the same manner as indicated in Fig. 3.

This is all well illustrated by comparing Fig. 2, which shows the relative stream lines through the blade system, and emphasizes the fact that the average deflection is smaller than the curvature of the blades, with Fig. 3, which shows the corresponding pressure distribution with varying pressure from the leading surface of one blade to the trailing surface of the other. Also in Fig. 3 there is plotted the relative velocities w, which are greater in the regions where the pressure is lower and vice versa.

Figs. 2 and 3 represent the conditions on the average surface of revolution DD' along which lies the paths of the filaments of flow and to which the cone CC' is tangential. It has been as-

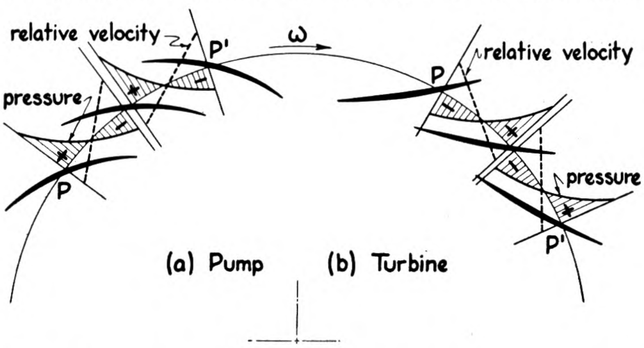


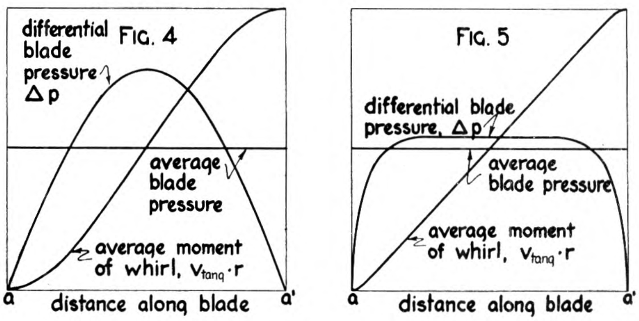
Fig. 3 Pressure and Relative Velocity Distribution for Blading Shown in Fig. 2

sumed that this surface of revolution can be replaced by the cone for the purpose of illustrating the conditions as existing in the flow. On every other such surface of revolution similar conditions exist, and so it is evident that the distribution of blade pressure is actually not uniform, as was assumed in Equation [8].

It is furthermore obvious that the pressure difference Δp at the leading and trailing edges must be zero, because such a difference can only exist when sustained by a material surface or body. Thus the blade pressure must drop to zero at the edges of the blades. In order to compensate for this reduction, below the average, the maximum values must naturally exceed the average. The actual distribution of blade pressure will be shown to depend upon how the flow exchanges its energy with mechanical energy during its passage between the blades. This exchange is determined by the rate of increase or decrease of moment of whirl ($v_{\text{tang}}\, r$), as shown by Equation [2], which is valid for intermediate partial distances between the outlet and inlet of the runner.

The value that has been taken for the moment of whirl ($v_{\text{tang}}\, r$) has been understood to be an average for all the filaments of flow in the space between two blades. It corresponds to the intensity and direction of the average relative flow which naturally is dependent upon the shape of the blades, especially the radius of curvature at any particular point along the contour. To illustrate this, in Figs. 4 and 5, two curves showing different types of variation in the average value of ($v_{\text{tang}}\, r$) and the corresponding distribution of blade pressure are plotted against the developed distance aa' of Fig. 1. Closer investigation shows that Δp is proportional to the slope of the curve representing ($v_{\text{tang}}\, r$). Although these diagrams are equally applicable for pumps or turbines, they do not intend to show the exact conditions for the shape of the blades illustrated in Figs. 2 and 3, but they do show how the distribution of Δp is affected by the rate of change of moment of whirl, which in turn is proportional to the rate of change of total energy H.

As before mentioned, even if the magnitude of Δp at each point of the blade is known, so far there is no way of determining how it is proportioned between surplus of pressure above the average on one side of the blades and deficiency below the average on the other. As a first approximation it could be assumed that it would be divided equally so that $\Delta p/2$ would represent both the surplus and the deficiency. Based on this assumption, and



Figs. 4 and 5 Relationship Between Change in Moment of Whirl and Distribution of Differential Blade Pressure

also allowing that there is a reasonably small deviation from uniform distribution of Δp over the entire projected area, it would be possible to design a blade of required properties with respect to the lowest pressure occurring in the blade system. It is this lowest pressure which is critical in determining under what conditions cavitation will take place. This idea will be discussed further in a later section. (See sections 7 and 9.)

4 Analogies to the Airfoil Theory

If we again consider the flow in that layer represented by aa' in Fig. 1, and as developed in Fig. 2, it is evident that each blade operates under conditions which are very similar to that of an airfoil or wing of an airplane. In fact, there is a strong resemblance between the cross-section of a blade and the shape of an airfoil. Furthermore, such a wing has a surplus of pressure on one side and a deficiency on the other, as indicated in Fig. 6. The side having the higher pressure is marked plus and the other minus. As a result of this distribution of pressure, there is produced an effect which in aeronautics is called "lift," but for blades it may better be considered as a thrust which is perpendicular to the direction of the approaching flow. The other component at right angles is called drag, or resistance to flow. In an ideal fluid the lift is the only reaction acting upon the wing or blade, but in a real fluid the drag component exists and must be taken into account.

On the high-pressure side of the profile the velocities are somewhat lower than on the low-pressure side. In Fig. 6 the stream lines have been drawn to indicate this fact by having the distance between each pair of lines so that there will be the same partial discharge. It will be seen furthermore that the stream lines approaching and leaving are deflected in the direction indicated by the two arrows by the effect of the pressure gradient around the edges.

A most significant difference between the behavior of a single wing and the blades of a turbine or pump is that in the case of the latter we must deal with a series of profiles arranged in circular

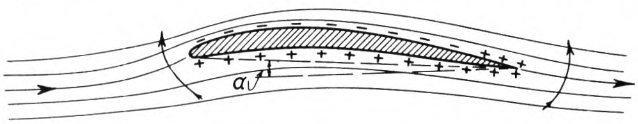


Fig. 6 Flow Along an Airfoil Profile

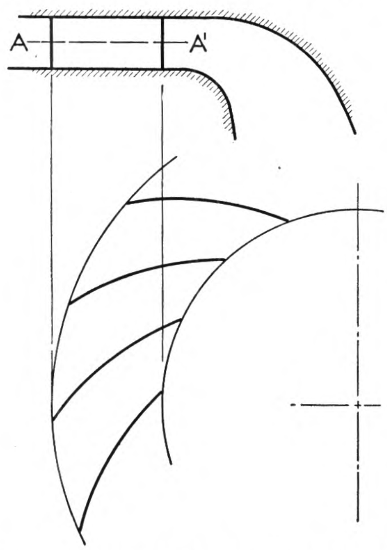


Fig. 7 Radial Arrangement of Blades

formation. It was seen in Figs. 1 and 2 that sections of the profiles cut by a layer of flow extend over circles of differing radius and the angle formed by an element of the cone, and the shaft may have any value from 90 deg to 0. It would be well to consider first the two extreme conditions:

(1) A radial arrangement of profiles is shown in Fig. 7. The surfaces of the blades are cylindrical, and the flow is bounded by two parallel planes normal to the shaft. For each plane parallel to the bounding surfaces, there are assumed to be the same conditions of flow, and furthermore, the flow is two-dimensional—i.e., it can be represented on a single plane. This arrangement is used for runners of high-pressure centrifugal pumps, and is useful for studying the influence of the number, shape, and angles of the blades upon the torque and head under different conditions of discharge and speed.(2) Such a series of blades may also be assumed to represent approximately the conditions of flow in any given layer, as in Figs. 1, 2, and 3.

(2) An axial(3) arrangement of blades is shown in Fig. 8, similar to Fig. 7, except that it has been drawn for an axial blade arrangement. The result of developing a flow layer is a straight and infinitely long series of profiles parallel to each other. Such a series may likewise be used to study the influence of the previously mentioned details by considering that the flow in the thin layer represented by the cylindrical section, having a thickness of dr, is identical with a similar section AA' in Fig. 7. In the axial runner, each cylindrical section is different from the others in angles, pitch, and shape of the blades. It is therefore necessary to investigate whether or not, and under what conditions, the flow in one layer influences that in the neighboring layers.

Whatever the result of such investigation may be, we could determine the torque in the shaft and, in consequence, the effective head absorbed or created by the runner, if we could only know the magnitude and direction of the resulting force caused by the reaction against the deflection of flow working upon each individual circular section. If, furthermore, we could obtain information from experiments made with profiles under simplified conditions, we possibly would have a good foundation for a theory of few-bladed axial runners.

A great many experiments have already been made for the purpose of investigating the airfoil theory. Wings or vanes having special profiles have been held in a fixed position opposed to a uniformly approaching flow. For a thin layer near the central plane normal to the width of the body and parallel to the approaching flow, the action can be considered to take place in a single plane, and the lift L and drag D may be computed from the experiments.(4)

It is obvious that conditions for the radial arrangement are so different that the results of such experiments cannot be transferred directly without investigating very closely the corrections to be applied. An attempt to make this transfer may not be worth while; it may be better to experiment in the first place with a radial arrangement of blades, or else to apply special theoretical calculations. Certainly in the case of the axial as contrasted with the radial runner, there is much more reason to apply directly, or with few corrections, experimental results and so get an approximation of the underlying theory. For this reason the next section gives a short outline of how such a theory may be applied.

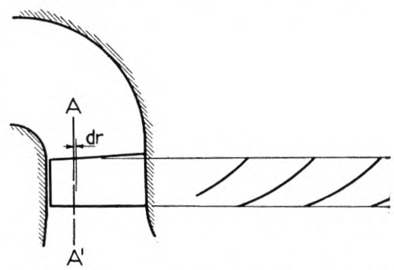


Fig. 8 Axial Arrangement of Blades

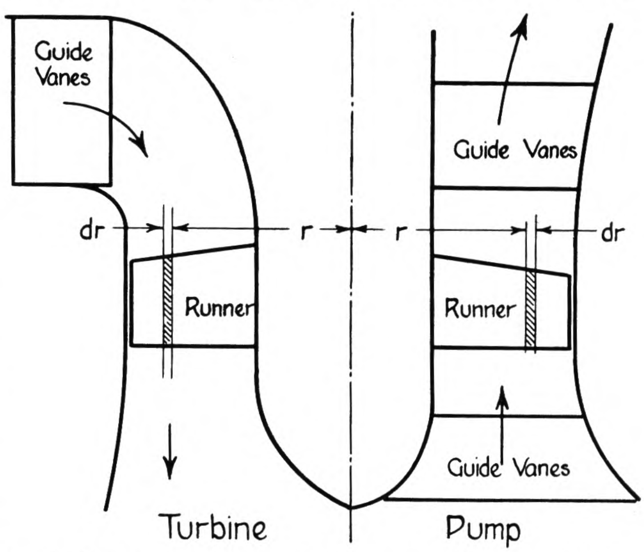


Fig. 9 Cylindrical Layers of Radius r and Thickness dr Used in Analysis of Theory of Axial Runners

5 Application of the Airfoil Theory to Runners of Propeller Pumps and Turbines(5)

If we treat, as suggested in Fig. 8, the thin cylindrical layer of flow, see Fig. 9, having an average radius r and a thickness dr

which is so small that uniform conditions may be safely assumed to exist across this incremental distance, the development of this layer may be represented by Fig. 10 for a pump and by Fig. 11 for a turbine. These diagrams show the cross-sections of the blade profiles and the relative flow which can be assumed to take place in this cylindrical section. In both cases the inclination of the chord of the blades relative to the direction of motion is given by the angle θ, while the distance from a point on one blade to the corresponding point on the next is given by the pitch S. The width of the blade is defined by the distance l between two perpendiculars to the chord. The height of the blading is given by X.

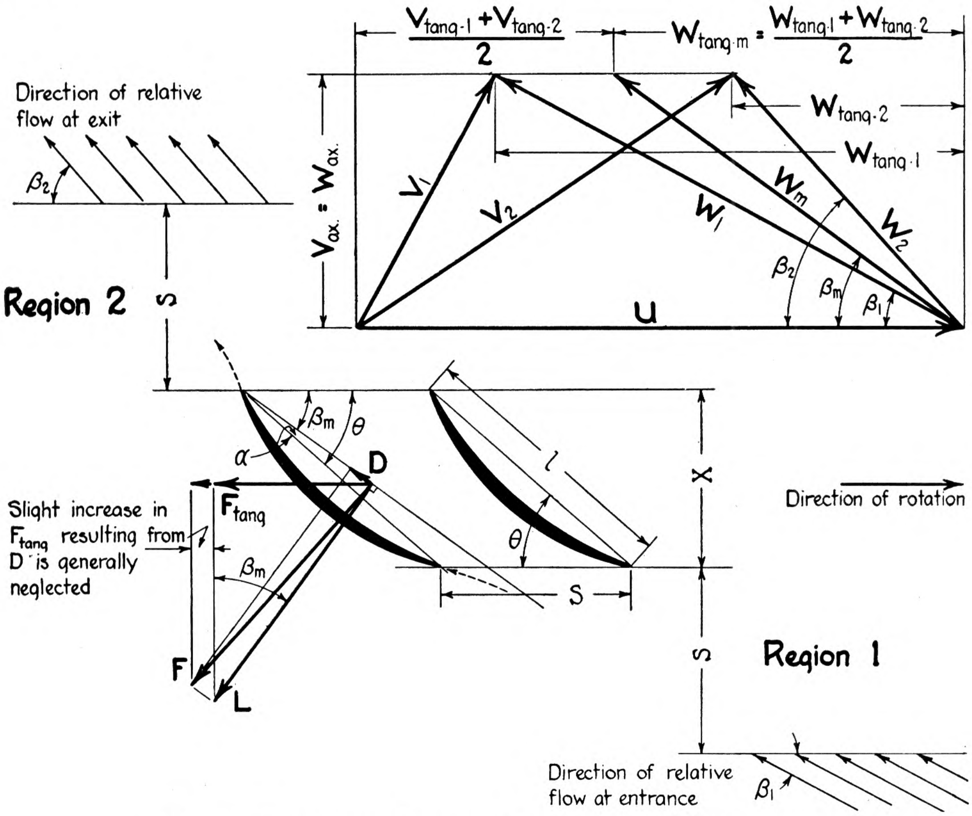


FIG. 10 BLADING AND VELOCITY DIAGRAM FOR AN AXIAL PUMP

The oncoming flow is assumed to be uniformly distributed and approaching at an angle β_1, relative to the direction the blade is moving until it enters region 1 at a distance of approximately S from the tips of the blades. The discharge is likewise considered to be uniform after passing through region 2 of like width, and leaves at an angle of β_2. These two angles correspond to those which would be required to satisfy the elementary theory. The effect exerted on the flow is therefore the same as would be exerted by a series of thin blades extending to include regions 1 and 2, and having an infinitesimally small pitch.

The tangential component of reaction between the flow and all the blades can in theory be computed from a knowledge of the forces acting on a single blade. These forces are determined experimentally and are usually stated in terms of the two components, lift and drag. Of course, when the same profile is mounted in a series, the conditions are altered, but for the ideal flow it may be shown theoretically that the lift is normal to the direction of the mean relative velocity w_m, which may be determined from the velocity diagrams as follows:

The axial components for all velocities w_1, w_2, v_1, v_2, w_m are all the same, since these are determined by the quantity of water passing and the discharge area, assuming uniform distribution over the entire area. It is customary to consider the tangential component of the mean relative velocity equal to the mean between the tangential components for w_1 and w_2—that is to say:

$$w_{\text{tang } m} = \frac{w_{\text{tang } 1} + w_{\text{tang } 2}}{2} \qquad [9]$$

Thus w_m may be determined in both magnitude and direction. The lift is taken as being normal to the direction of the average relative velocity, while the drag is considered to be parallel to it. In this situation this average relative velocity takes the place of the approach velocity in experiments with a single profile,(6) and so L acts in a direction perpendicular to it. It will be seen on Figs. 10 and 11 that β_m differs from θ by a small angle α. It is furthermore evident that if L and D are known, the resulting force F can be found, and from it the tangential component F_{tang} can be determined.

Experimental results give the desired values of L and D as functions of this angle α, which is defined as the difference between the direction of the approaching velocity and the chord of the profile and corresponds to the angle of attack.(7)

According to the airfoil theory, the lift and the drag are given by

$$L = \zeta_l A \gamma \frac{v_0^2}{2g} \qquad [10]$$

$$D = \zeta_d A \gamma \frac{v_0^2}{2g} \qquad [11]$$

where A is the area, which is equal to the length l times the distance b, between the parallel walls of the testing tunnel or flume; v_0 is undisturbed velocity of approach, and ζ_l and ζ_d, the lift and drag coefficients, respectively, are determined experimentally as functions of α. There is one value of α for which the ratio of drag to lift is a minimum, and which will be seen to be desirable to approximate as far as possible in the selection of blade-design data.

In the case under consideration, the area is the product of ldr, and as w_m corresponds to v_0, appropriate substitutions in the foregoing equations give

$$dL = \zeta_l \, l \, dr \, \gamma \frac{w_m^2}{2g} \qquad [12]$$

$$dD = \zeta_d \, l \, dr \, \gamma \frac{w_m^2}{2g} \qquad [13]$$

In order to make the formulas developed in the course of this reasoning the same for turbines and pumps, let us introduce the subscripts $_p$ for pressure side and $_s$ for suction side in place of $_1$ and $_2$ as used previously for entrance and exit.

To cover a general case rather than the conventional design, it will not be assumed that $(v_{\text{tang}}\, r)_s$ is zero, because it is very possible that in the future it will be found desirable to have positive or even negative values of moment of whirl on the suction side of the runner, even though such practise might require a guide apparatus or similar device at the intake of a pump. It should be repeated that in present practise the $v_{\mathrm{tang}\ p}$ of a pump is determined by the outlet conditions, and for a turbine by the guide vanes, whereas $v_{\mathrm{tang}\ s}$ is governed by the inlet conditions of a pump and the outlet conditions of a turbine.

The effect of drag was shown by Figs. 10 and 11 to increase the tangential component in a pump and to reduce it in the case of a turbine. Now in order to avoid complication and because D is generally but 1 or 2 per cent of the lift, it is allowable to neglect it when computing this tangential component, but to take it into account when calculating the losses and the efficiency.

It will be seen from the geometrical construction that neglecting the slight effect of drag

$$dF_{\mathrm{tang}} = \zeta_l\ l\ dr\ \gamma\ \frac{w_m^2}{2g}\sin\beta_m \qquad \text{........[14]}$$

It is furthermore apparent from the velocity diagram of these same figures that

$$w_m^2 = v_{\mathrm{ax}}^2 + \left(u - \frac{v_{\mathrm{tang}\ s} + v_{\mathrm{tang}\ p}}{2}\right)^2 \qquad \text{........[15]}$$

and

$$\sin\beta_m = v_{\mathrm{ax}}/w_m \qquad \text{....[16]}$$

By substituting v_{ax}/w_m for $\sin\beta_m$ in [14], it follows

$$dF_{\mathrm{tang}} = \zeta_l\ l\ dr\ \frac{\gamma}{2g}\ v_{\mathrm{ax}}\ w_m \qquad \text{........[17]}$$

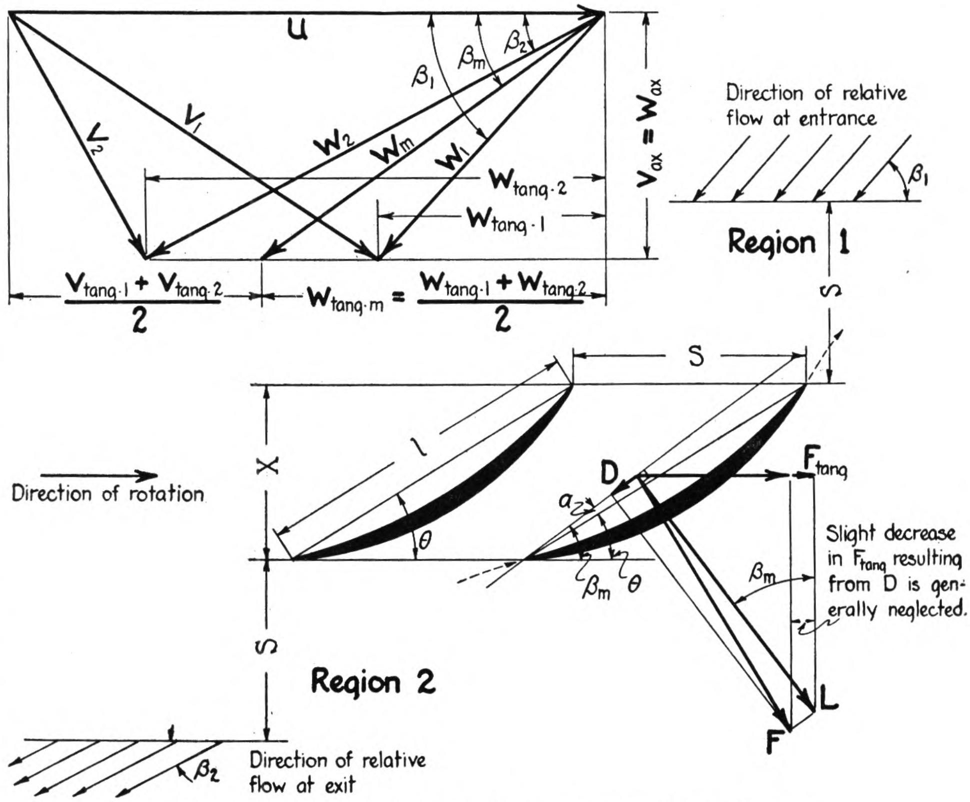


FIG. 11 BLADING AND VELOCITY DIAGRAM FOR AN AXIAL TURBINE

Taking the square root of [15] and substituting it for w_m in [17],

$$dF_{\mathrm{tang}} = \zeta_l\, l\, dr\, \frac{\gamma}{2g}\ v_{\mathrm{ax}} \sqrt{v_{\mathrm{ax}}^2 + \left(u - \frac{v_{\mathrm{tang}\ s} + v_{\mathrm{tang}\ p}}{2}\right)^2} \qquad \text{..[18]}$$

Since force multiplied by velocity is power, which in turn is equal to the quantity of water in pounds times the effective head in feet, $F_{\mathrm{tang}}\,u$ would equal the effective head $\Delta_1{}^2H$ times W. W is equal to the density γ in pounds per cubic feet times Q in cubic feet per second, which is itself the product of the axial velocity v_{ax} and the discharge area Sdr. Consequently there is built up

$$dF_{\mathrm{tang}}\, u = v_{\mathrm{ax}}\, S\, dr\, \gamma\, \Delta_1{}^2H \qquad \text{............[19]}$$

But $\Delta_1{}^2H$ being previously shown to be $u(v_{\mathrm{tang}\ p} - v_{\mathrm{tang}\ s})/g$, a substitution gives

$$dF_{\mathrm{tang}}\, u = v_{\mathrm{ax}}\, S\, dr\, \gamma\, u\,(v_{\mathrm{tang}\ p} - v_{\mathrm{tang}\ s})/g \qquad \text{.........[20]}$$

Either side of this equation expresses the foot-pounds of work per second done or absorbed by the flow passing through the space between two adjacent blades. Now combining [18] and [20] and canceling v_{ax} outside the radical, as well as dr, γ, and g, we get

$$\frac{\zeta_l\, l}{2}\sqrt{v_{\mathrm{ax}}^2 + \left(u - \frac{v_{\mathrm{tang}\ s} + v_{\mathrm{tang}\ p}}{2}\right)^2} = S(v_{\mathrm{tang}\ p} - v_{\mathrm{tang}\ s})$$

which may be simplified to

$$\frac{\zeta_l\, l}{2S} = \frac{v_{\mathrm{tang}\ p} - v_{\mathrm{tang}\ s}}{\sqrt{v_{\mathrm{ax}}^2 + \left(u - \frac{v_{\mathrm{tang}\ s} + v_{\mathrm{tang}\ p}}{2}\right)^2}} \qquad \text{......[21]}$$

This equation has been written purposely in such a form that the four values which may be considered as known will be on the right-hand side, as these must be selected in advance. (Remarks concerning this selection will be contained in section 9.) It appears therefore that there are three unknowns, the combination of which must be constant.

Assume first that a profile, the properties of which are known, is chosen, and it would be advantageous to select first the value of ζ_l that would give a minimum ratio ζ_d/ζ_l, thus leaving but two variables. There are only a limited number of possible values for S, as $2\pi r/S = z$, which must be an integer, so l remains to be determined by trying various combinations until a convenient length of blade is found, and then

$$\theta = \beta_m + \alpha \qquad \text{..................[22]}$$

all of the values are known which are necessary for the design of the profile for this circular section. The subsequent sections are analyzed in the same way. In so doing it is possible to change the shape and dimensions of the profile in the several successive sections, or else to hold to one profile and change only the dimensions to satisfy Equation [21]. Efficiency and strength of the blades, together with danger of cavitation, all have to be taken into account as governing the selection of these variables.

It is now necessary to look further to be assured against cavitation. It may prove necessary to revise the initial choice of ζ_l by altering α in spite of sacrifice in efficiency. This subject will be discussed more in detail in section 9.

6 Criticism of the Airfoil Theory

In the preceding section there has been discussed the utilization of experimental results on one profile for the design of a turbine or pump. It should next be considered whether or not such results obtained from a single profile are applicable to a series of blades, particularly when arranged like the spokes of a wheel. A number of theoretical investigations have been made in which the fluid has been considered as ideal, but because of difficulties in the mathematics involved, such studies cannot take into account all the details such as curvature, thickness, etc., and especially the drag caused by friction. Consequently, the problem should be looked at from the material point of view in order to see what can be expected in practise.

It has been shown in Fig. 6 that when a profile is exposed to an approaching flow, there is a definite pressure distribution. In the beginning we were concerned simply with the distribution at the surface, because it is this alone that determines the lift and the drag. To look further and study the effect of mutual interference of several profiles upon each other, it is necessary to remember that the elements of pressure increase or decrease at the surface are radiating out into the flow. This influence does not extend very far as long as the deflecting effect is small, and in such cases the disturbances in the pressure field caused by each individual blade do not interfere.

However, there is another very marked effect which results from the narrowing of the passages between the blades. All of the velocities in the spaces between the blades are increased, with consequent general lowering of pressure. Thus the pressures that would otherwise be high are decreased, and the low pressures on the other side of the blade are still further reduced. The effect of this change upon the lift cannot be predicted without detailed study of the shape of the blade, but Fig. 12 shows what happens in a specific case where the wings of a biplane are exposed to an approaching flow with small angle of attack. The original pressure distribution for the two wings each taken separately is indicated by the solid pluses and minuses, while the influence of the combination of the two is denoted by the broken minuses, - -. It will be seen that the net effect is that the lift of the upper wing is decreased, whereas for the lower it is increased. It has actually been found by experiments that the lower wing carries the greater part of the weight of the ship.

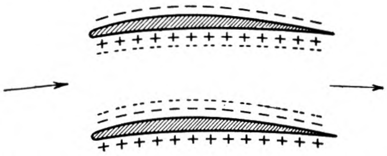


Fig. 12 Mutual Interference Between Wings of a Biplane

For higher angles of attack this effect is less marked as it becomes more and more obliterated by the increasing magnitude of the pressures set up by the wings themselves. It is furthermore evident that this influence is affected by the relative positions of the profiles themselves, which may be set farther apart or nearer together or shifted longitudinally with respect to each other. As a rule for all angles of attack, profiles arranged as shown in Fig. 12 have a total lift less than the sum of the lifts of the individual wings.

The principal difference between the wings of a biplane and the blades of a turbine or pump is that, on account of being arranged symmetrically about the axis, the tangential thrust and the drag for each must be the same. It is now a question as to whether the forces acting upon a single blade of a series are greater or less than those working upon a single blade alone exposed to the same conditions of flow. The mathematical investigations previously mentioned(8) unfortunately cannot take into consideration all of the details which are so essential to this problem. Such studies are only carried out for a parallel series of blades, and the results so derived are valid for but one circular section of the actual runner. If there is built up a special runner composed of a number of parallel series, each of them satisfying the design conditions for the corresponding cylindrical section, then two problems must be encountered: The first is to determine the correct position of the established cylindrical sections with respect to each other to be sure that the pressure distribution along the several contours will correspond to each other and not cause cross-flow from one section to another. The second is to see whether after all this desired result is possible with the profiles that have been selected.

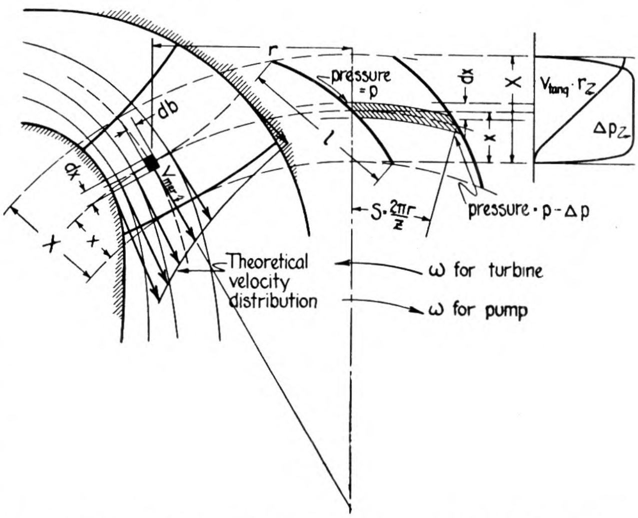


Fig. 13 Theoretical Velocity Distribution and Developed Layer of Flow for a General Type of Runner

Difficulties do not end here, for the mutual interference of a series of blades in parallel arrangement is certainly different from that of the same profiles in radial arrangement constantly changing in contour from one section to another throughout the distance from periphery to the hub.

It will thus be appreciated that there are many unknown factors in the direct application of airfoil theory and experimental knowledge to the design of turbines and pumps. Laboratory experience with such runners must certainly be called upon to check the results of the use of such a theory, and therefore it may be questioned whether or not the airfoil theory is, after all, the best one upon which to base our ideas.

In the following section there is given another point of view which is based more nearly upon the elementary theory discussed at the outset.

7 Another Theory for Runner Design

The fundamental assumption in the airfoil theory of design is that similar conditions exist for a single blade exposed to approaching flow as exist in the blading system of a runner. Another way of looking at the problem would be to begin with a consideration of a theoretically perfect system having an infinite number of thin blades, and then study the corrections that are necessary when the number of blades is reduced to a finite and finally to a very small number, while at the same time their thickness is correspondingly increased. The blades are initially taken as geometrical surfaces, representing the relative flow at any point in the space occupied by the runner. As such a method of establishing a logical theory, susceptible to check by progressive experimentation, is valid for all types of runners, there has been selected for illustration the general type shown in Fig. 13.

The same conventions in regard to subscripts $_p$ and $_s$ will be observed as explained in section 5. It should be remembered that this is a discussion of a general case, and so $(v_{\text{tang}}\, r)_s$ is not assumed to equal zero.

The first step is to consider how free flow would pass through the annular spaces between the hub and the casing if there were no blading at all. The free flow may be considered to follow along meridians of surfaces of revolution, and the velocities therefore are designated as v_{mer}. The theory of an ideal flow allows us to figure out the velocity distribution, taking into account the effect of centrifugal force caused by the curvature of the stream lines.(9) Fig. 13 shows also the velocity distribution in one of the cross-sections of this annular passage. This velocity curve should next be corrected for the effect of skin friction and turbulence.[3] So far this can be done only by comparison with experimental results in similar cases. The author has started systematic tests in his laboratory at Karlsruhe to study the velocities through annular passages in a region of turbulent flow. Fig. 14 shows such a diagram for four straight pipes having the same diameter, but three of them having cores of different diameters. These results can be applied to propeller turbines, and the axial flow bounded by the throat ring and hub, although theoretically uniform, should be corrected in the light of these experiments.

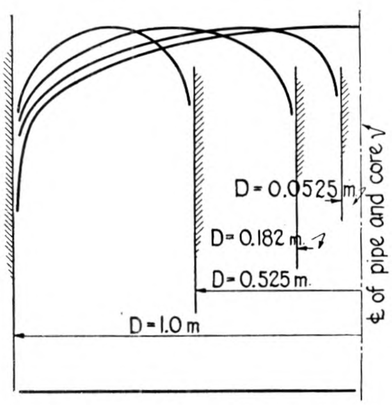


FIG. 14 ACTUAL VELOCITY DISTRIBUTION IN PIPE WITH CORES OF THREE DIFFERENT DIAMETERS

Fig. 15 shows a corresponding diagram of velocities actually measured in an annular passage used for centrifugal pumps. The velocities for an ideal fluid are plotted in dash lines to show the contrast with the actual in solid lines. The change of shape of the curves for different cross-sections depends upon whether the flow is being accelerated or retarded.

Such tests should be made to cover a sufficiently wide range of conditions, and a theory should be evolved which would enable us to predetermine the velocity distribution in actual installations. So far it is necessary to depend upon estimates and judgment.

It follows that if the velocity distribution is altered, then the stream lines would have to be revised so that there will always be the same partial discharge between each adjacent pair of lines.

Based upon knowledge of stream lines for free flow, the next step is to isolate a thin layer between two stream lines. The velocity v_{mer} along the distance X measuring the length of the layer along a meridian and beginning at the suction edge of the blading is now known.

In order to study the flow inside this layer of revolution, the tangent cone may be resorted to and developed by unrolling, as in the right-hand part of Fig. 13. Now the overall increase in $(v_{\text{tang}}\, r)$ is given from fundamental considerations, and $(v_{\text{tang}}\, r)_s$, whatever its value may be, corresponds to $x = 0$, while $(v_{\text{tang}}\, r)_p$ corresponds to $x = X$ and may be so plotted. In the meantime $v_{\text{tang}}\, r$ is a function of x, and the law by which one varies with respect to the other may be selected arbitrarily, but the consequences of such selection must be investigated.

Now if the number of blades is not infinitely high, but still very high, the variations in pressure and velocity along the circle between them will be correspondingly very small, and it is allowable to make a simple but reasonable assumption in this regard. This is that the pressure varies linearly over the distance between the blades and that the variation in velocities can still be neglected. Now the momentum theory can be applied to that incremental volume having the form of a prism extending between the front and back surfaces of two adjacent blades, in exactly the same way in which it was applied to the average flow at the outset of this discussion.

The cross-section as shown in Fig. 13 is the area $dxdb$, and the length is S, or $2\pi r/z$.

By following the same reasoning as in the development of Equation [1], we get two expressions for the small part of the torque balanced by the reaction of the flow during its passage through that volume:

$$dT = \frac{d}{dx}\,(Fr)\,dx = \frac{dW}{g}\,\frac{d}{dx}\,(v_{\text{tang}}\, r)\,dx \ldots\ldots\ldots [23]$$

$$= v_{\text{mer}}\, S\, db\,\frac{\gamma}{g}\,\frac{d}{dx}\,(v_{\text{tang}}\, r)\,dx$$

The transposition from one expression to another shows that $v_{\text{mer}}\, Sdb$, the incremental discharge, times the density γ is the incremental weight, the deflection of which corresponds to the incremental change in the tangential velocity $\frac{d}{dx}\,(v_{\text{tang}}\, r)\,dx$.

Likewise in the same manner that Equation [8] was developed

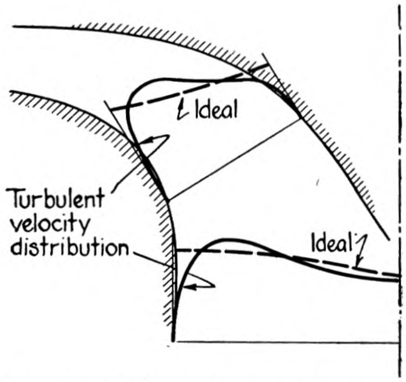


FIG. 15 COMPARISON OF THE IDEAL AND TURBULENT VELOCITY DISTRIBUTIONS IN THE ANNULAR PASSAGE OF A PUMP

$$dT = \pm db\,dx\,r\,\Delta_0^s p \ldots\ldots\ldots\ldots\ldots\ldots [24]$$

By combining [23] and [24], the result is:

$$\Delta_0^s p = \pm\,\frac{2\pi\, v_{\text{mer}}\,\gamma}{z\,g}\,\frac{d}{dx}\,(v_{\text{tang}}\, r) \ldots\ldots\ldots\ldots [25]$$

The term $\frac{d}{dx}(v_{\text{tang}}\, r)$ is a measure of the rate of increase of $(v_{\text{tang}}\, r)$ along x. If the relation between $v_{\text{tang}}\, r$ and x is plotted, then the slope of this curve gives $\frac{d}{dx}(v_{\text{tang}}\, r)$. It is necessary to keep in mind, as brought out in section 3, that Δp at the edges of the blades must be zero. When the number of blades is infinite, $S = 0$, and so Δp is indeterminate from this equation, but we are not interested in this case. When, on the other hand, the number of blades is finite, then Equation [25] shows that $\frac{d}{dx}\,(v_{\text{tang}}\, r)$ must be zero at the edges of the blades. Therefore,

[3] The expression "turbulence" is considered here as the property which distinguishes a flow from "laminar" or "streamline flow." In a turbulent flow the individual fluid particles change from one stream line to another.

whatever may be the relation between x and $(v_{\text{tang}}\, r)$, the curve must have a slope of zero at the end-points and consequently be S-shaped.

On the other hand, the average pressure between the blades can be computed by an equation which corresponds to Equation [7].

If we consider H, the specific energy which is increased by transfer to the flow from the blades, to be a function of x and designate by H_s the value at the suction side where $x = 0$ and by H_x the value at any intermediate point at distance x from the entrance, then we can write

$$H_x - H_s = \frac{\omega}{g}\left\{(v_{\text{tang}}\, r)_x - (v_{\text{tang}}\, r)_s\right\} \quad \text{[26]}$$

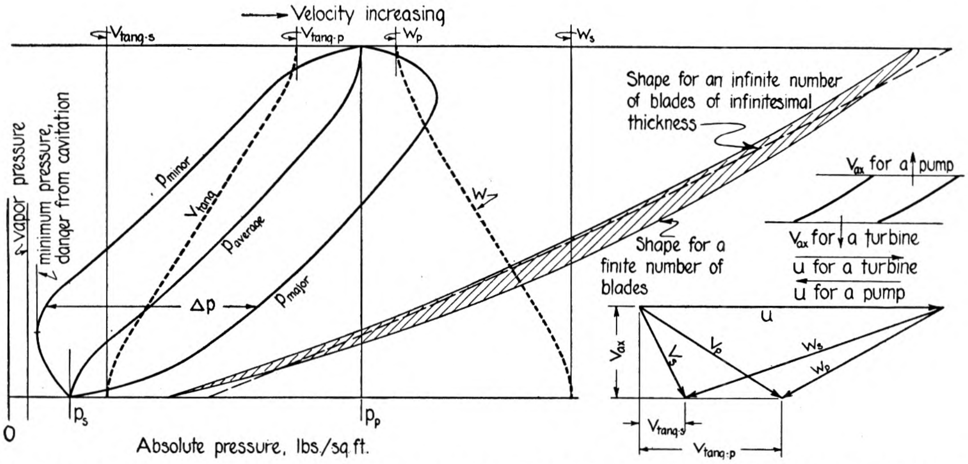


FIG. 16 RELATIVE VELOCITY, TANGENTIAL COMPONENT OF WHIRL, AND PRESSURES IN CYLINDRICAL SECTION OF AXIAL RUNNER PLOTTED AGAINST DISTANCE ALONG BLADE

(Together with profile that would be used for runner having a few blades compared with shape for infinite number of blades.)

But by definition

$$H = \frac{p}{\gamma} + h + \frac{v^2}{2g} \quad \text{[27]}$$

therefore

$$H_x - H_s = \frac{p_x - p_s}{\gamma} + h_x - h_s + \frac{v_x^2 - v_s^2}{2g} \quad \text{[28]}$$

Combining Equations [26] and [28] and transposing, gives

$$\frac{p_x - p_s}{\gamma} = \frac{\omega}{g}\left\{(v_{\text{tang}}\, r)_x - (v_{\text{tang}}\, r)_s\right\} - \frac{v_x^2 - v_s^2}{2g} - (h_x - h_s) \quad \text{[29]}$$

It now appears that if $(v_{\text{tang}}\, r)_x$ is a function of x, then the value corresponding to any value of x can be found. The moment of whirl $(v_{\text{tang}}\, r)$ is determined by the fundamental data. The r's being fixed by the physical layout, both v_{tang}'s may be determined, and from them the v's, since $v^2 = v_{\text{tang}}^2 + v_{\text{mer}}^2$. The h's also are determined by the height above a datum plane, so that with p_s known, p_x may be computed for all values of x.

In the case of a large number of blades the pressure may be considered to vary uniformly along the arcs connecting the corresponding parts of two adjacent blades. In other words, one-half the pressure difference Δp is a drop below the average pressure and the other half an increase above it. Thus we are in a position to calculate the pressure on the low-pressure side of the blade, p_{min}, since

$$\frac{p_{\text{min}}}{\gamma} = \frac{p_x}{\gamma} - \frac{\Delta p}{2\gamma} \quad \text{or} \quad p_{\text{min}} = \frac{2p_x - \Delta p}{2} \quad \text{[30]}$$

Substituting for $(p_x - p_s)/\gamma$ the expression given by Equation [29] and for $\Delta p/2\gamma$ the expression according to Equation [25],

$$\frac{p_{\text{min}}}{\gamma} = \frac{p_s}{\gamma} + \frac{\omega}{g}\left\{(v_{\text{tang}}\, r)_x - (v_{\text{tang}}\, r)_s\right\} - \frac{v_x^2 - v_s^2}{2g} - (h_x - h_s) - \frac{S\, v_{\text{mer}}}{2\,r\,g}\,\frac{d}{dx}(v_{\text{tang}}\, r) \quad \text{[31]}$$

The foregoing may be worked out and the results presented graphically, but to illustrate the principles involved it is sufficient to show simply the case of an axial runner. Fig. 16 gives the plots of all the functions. In the general case it is necessary to consider the changes in r and u between the pressure and suction edges, and therefore the velocity diagrams which follow having a common line for u apply only to axial runners. This diagram applies for turbines as well as for pumps, simply by reversing the directions of flow and rotation as indicated and by properly shaping the leading and trailing edges. It is important to note in this connection that even with many blades it is possible and almost inevitable that the lowest pressure on some part of the blade will be below that of the suction pressure. The diagram indicates that it is this point that is critical in determining whether cavitation will take place or not.

If the number of blades is made progressively smaller, the assumptions introduced so far are no longer true and the method must be improved. Nevertheless, as far as we have gone, Equation [31] shows that the pressures depend to a large extent upon the rate of change in the moment of whirl produced by the blades themselves. If in this Equation [31] the values $(v_{\text{tang}}\, r)$ and $\frac{d}{dx}(v_{\text{tang}}\, r)$ are taken as average values, it is still valid even for comparatively few blades according to the momentum theory.

There is now before us the same problem that appeared in the case of the application of the airfoil theory. If the $(v_{\text{tang}}\, r)$ is fixed as a function of x in one layer, how is it to be determined upon for adjacent layers? It is evident that this cannot be done arbitrarily. The general law which must be followed is that in the entire blading system the equation

$$\frac{p}{\gamma} + h + \frac{w^2 - u^2}{2g} = \text{constant} \quad \text{[32]}$$

must be fulfilled. This equation, which was developed with particular reference to ideal flow, can also be applied to turbulent flow. The reason why this result is not secured at first is that the distribution of v_m is first fixed, and the choice of $(v_{\text{tang}}\, r)$ together with the fixed v_m determines the absolute velocities v and the relative velocities w. This restriction applies only for average values between the blades of multi-bladed runners. In the case of a few blades it is necessary not only to compare the average values of several layers, but also to compare the conditions varying along the parallels.

Again assumptions have to be introduced to simulate more closely the actual conditions. For instance, it may be assumed that over the distance which separates two blades the relative velocity is varying in a straight-line relation, which means that the pressure is varying with some higher power. A further assumption is that the angle that the relative velocity makes with the tangential direction of a point on the runner varies together with the magnitude of this velocity.

To allow for these points which are not readily susceptible to analysis, simple and reasonable assumptions can be introduced as the result of experimentation carried out with special forms of blades arranged either in a series or singly, or perhaps more exact calculations. By such successive approximations a method can be developed, the final purpose of which is to establish a basis for the selection of the relation between ($v_{\text{tang}}\, r$) and x, and to carry out a detailed determination of velocities and pressures between the blades as well as upon their surfaces.

It is necessary, for two reasons, to know the distribution of velocities and pressures. In the first place, Equation [32] must be satisfied, for if it is not, there will be secondary flows across the blades which will lead to additional complications arising from the need of considering their radial and axial components. In order to appreciate this more fully, it is desirable to consider the conditions existing for a straight series of parallel blades arranged between two parallel and plane bounding surfaces, as shown at the top of Fig. 17.

Suppose, first, that the blades are identical cylindrical surfaces, which means that they have the same profile in each layer parallel to the bounding planes, and suppose, furthermore, that the flow approaches this blading system uniformly in all of these parallel planes. This means that between the blades the flow also takes place in a plane, and the pressure along each straight line perpendicular to the planes is uniform. Consequently, there would be no reason why secondary flow across the blade should exist. The variations in v_{tang}, which here is analogous to the $v_{\text{tang}}\, r$ in the case of runners, are necessarily the same for all positions along this perpendicular.

Imagine, on the other hand, that the blades have different profiles, as shown in the lower portion of this Fig. 17. Then the distribution of pressure along the perpendicular obviously can no longer be uniform, since there would be a pressure gradient from one point to another. Consequently, a flow along the perpendicular is sure to take place. The variation of v_{tang} is no longer uniform across the blades, and is therefore analogous to the conditions which exist in a turbine or pump runner where the blades do not satisfy Equation [32] because of improperly selected distribution of $v_{\text{tang}}\, r$.

A second reason for knowing the pressure inside the blading system is because it is essential to know not only the lowest pressure, but its location. This is necessary in order to predict the maximum conditions of head or discharge with which it would be possible to operate the pump or turbine without danger of cavitation, as cavitation will take place when the minimum pressure approaches the vapor pressure of the water, as indicated in Fig. 16.

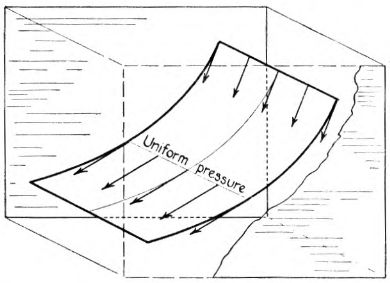
There now remains to be determined the shape which the blades take corresponding to a given distribution of $v_{\text{tang}}\, r$. For that purpose again consider a single layer and suppose the number of blades to be very high and the blades themselves to be very thin. Under these assumptions, the relative flow in the layer follows the curves which are identical to the intersection of the blades with the surfaces of revolution forming the layers; we can then write (see velocity diagram in Fig. 18):

$$v_{\text{tang}} = u - v_{\text{mer}} \cot \beta \qquad [33]$$

from which there follows by simple transposition $\cot \beta = (u - v_{\text{tang}})/v_{\text{mer}}$. The angle β is considered to be positive and always less than 90 deg.

The fraction on the right side is a known function of x, inasmuch as $v_{\text{tang}}\, r$ is known from the fundamental assumptions. Therefore, cot β and β itself become functions of x for this layer. From this relation the profile of the blade in this layer can be determined by some graphical method, and in the end the surface of the blade is represented by the intersection of the curves with all layers.

From this arises the main problem: If there are but a few blades, how must they be curved to force upon the flow the same average deflection as assumed by the choice of the relation between $v_{\text{tang}}\, r$ and x? An offhand answer is that they must be curved *more* than determined by the foregoing calculation. That means that the blade angle must be exaggerated at the leading edge to compensate for the bending effect of pressure described in sections 3 and 4 and Figs. 2(a) and (b) and 6. At the trailing edge this exaggeration has also to be made to overcome the similar bending effect which would otherwise reduce the total deflecting effect of the blades. The amount of the correction which must be applied for comparatively few blades as compared with that for many, must as yet be estimated. In Fig. 16 a comparison is shown between the shape which would be adopted for infinitely many thin blades and the one that would be used for a few blades of a definite thickness. The infinitesimally thin blade in this figure has been plotted to correspond with the variation of v_{tang} with x, as shown in this same figure.



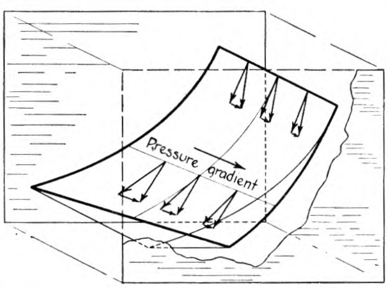


FIG. 17 CROSS-FLOW INDUCED BY PRESSURE GRADIENT ACROSS BLADE

8 ADAPTATION OF BOTH THEORIES TO ACQUIRING PROGRESSIVE EXPERIENCE AND KNOWLEDGE

Obviously it is prudent to check any theory of runner design by model test. If such tests do not give results in agreement with the theory, it is first hard to find out whether it is the theory as a whole or only the details that are incorrect and then just which details these may be. On the other hand, if the results agree with the theory with respect to unit discharge, unit speed, and efficiency, we only know that the composite effect is satisfactory while the details may simply mutually compensate. Of course, a comprehensive test made with models, especially with prototypes, would lead to a more profound knowledge. For example, to measure the distribution of pressure on revolving blades would help very much to compare the theoretical with the actual conditions. With small models it is not easy to take such readings on the revolving runner. With prototype machines, for instance on large propeller runners, it should be much easier, because of the actual dimensions of the blades, hub, and shaft, to allow the arrangement of piezometer holes connected through the hub and hollow shaft by the means of small tubes coming out at the end of the shaft. Even then it would be decidedly difficult to carry out, either with models or full-size machines, the complete system of tests which would be necessary to develop a reliable theory for turbine and pump design—that is, reliable in the sense that it could be adapted for all conditions and secure the highest efficiency and safety against cavitation.

It may not be expected that such a theory would result in still higher efficiency than those that have been obtained already, for example, 92 per cent in Europe at the Ryburg Schwörstadt plant on the Upper Rhine, but a more perfect theory would give assurance against mistakes, failures, and disappointments. It follows that because it is impossible to run a systematically arranged series of tests on turbines or turbine models, for the purpose of checking all the details underlying any theory, it is necessary to make detailed tests under simplified conditions. Undoubtedly a combination of both theories as previously described will prove the best foundation for runner design. Therefore, experimental devices, based on the ideas underlying both of these, will be needed. For example, the resulting forces exerted upon a series of profiles in parallel or even radial arrangement should be measured to determine the lift, the drag, and the velocity and pressure distribution under various angles of approach.[4]

Such tests should be made with profiles which are intended to be used for runner blading, and the difference of the effect upon the profiles when arranged in parallel or radial series should be determined; in this way the mutual interference can be studied. This is one of the two main problems in turbine design.

The other is to form a blade, having blade pressure distribution Δp as uniform as possible. The success of any effort in this line can be verified by taking readings of pressures along the blades.(10)

To analyze the results of tests of these two types, either the airfoil theory or the theory developed from infinitely many blades may be used, but in either case both must then be corrected and adapted to the conditions under which the profiles are used in the runners. This must be accomplished by actual tests themselves, for performance cannot be predicted in detail.

Other incidental problems, not so essential as those mentioned, but still important, should be studied in like manner. For example, one question is how the product $v_{\text{tang}}\,r$ generated by the guide vanes influences the distribution for free flow. In this case the guide vanes overhanging the curb ring cannot give the flow a constant moment of whirl if the distribution of velocity v_{mer} remains the same as for free flow. An alteration in the natural moment of whirl causes a pressure distribution which is not conducive to the same velocity distribution as for free flow. Hence the conclusion must be drawn that guide vanes overhanging the curb ring change the distribution from that for free flow, and therefore must be taken into account in the design of the runner. The effect can be studied mathematically, but the results should be checked by experiment made with guide vanes arranged only in circular or parallel series and with and without overhang. The velocity distribution in three dimensions should be observed for such flows.(11)

If it is necessary to arrange the guide vanes overhanging the curb ring in order to reduce the over-all diameter of the distributer, then there are three questions. Is it necessary to deform the lower parts of the guide vanes in order to get constant $v_{\text{tang}}\,r$? Is it possible, or perhaps better, to allow some difference in $v_{\text{tang}}\,r$ and to adapt runner blades to that distribution?

[4] A large number of tests have been made in Germany. The author, in his laboratory at Karlsruhe, has tests now under way in order to compare the effect of several profiles when arranged in parallel, with the effect of the same profiles when arranged as turbine runners. A close mathematical investigation is being made at the same time, and attention is called to a very interesting study, both mathematical and experimental, by Busmann.

Finally, is it worth while, after all, to worry about this discrepancy? Again, in this case, preliminary tests with parallel series of blades can be made to study the fundamental principles before the turbine as a whole has been designed and tested.

These examples show on the whole that the study of modern hydrodynamics offers a wide possibility of assisting in the search for a logical theory of turbine design, especially in the field of high specific speed. This does not require in every case tests with entire turbines or pumps, but often detailed problems can be studied by simplified set-ups, bringing out the essential points in a more distinct way than is possible even with complete tests.[5]

Naturally, investigations using the remarkable methods that have been very successful in the development of airfoil theory should be employed in such experimental research work.

One thing is certain: If we want to operate our equipment safely, or to extend the development to even higher speeds, we cannot depend solely on the experience of yesterday. We must use all weapons of modern hydrodynamics in its experimental and mathematical branches, in order to solve these problems.

9 General Discussion of Future Tendencies in the Design of High-Speed Runners With Regard to Efficiency and Cavitation

All of the foregoing paragraphs have dealt with the problem of how to develop a blading system providing the difference of the moment of whirl or $(v_{\text{tang}}\,r)$ is given by its relation to the net or effective head. Taking a more general viewpoint, we have assumed that in the future there will be a demand to determine in advance and control the whirl moment at both pressure and suction side of the runner.

There is now to be considered what should be the selection for moments of whirl on the two sides of the runner to suit a given specific speed. Although the answer to this problem depends to some extent upon the details that have been discussed in the foregoing sections, it is well to emphasize some of the essential elements in the fundamental layout of the velocity diagram and their effect on specific speed.

(*a*) *Mathematical Background.*

As a starting point it is necessary to have before us a concrete conception of specific speed. By definition it is the number of revolutions for a homologous machine operating at 1 ft head and developing 1 hp[6] and may be expressed

$$n_s = n\,\frac{HP^{1/2}}{H_t^{3/4}} \qquad [34]$$

where n_s is the specific speed, n the number of revolutions for the actual machine, HP is the horsepower, and H_t the head, which will be used as a shorthand expression for $H_p - H_s$.

Although this formula is very commonly used, its significance would be more appreciated if engineers would give consideration as to how it is derived. In an attempt to clear away any feeling that this expression is mysteriously evolved, let us imagine a given machine turning n revolutions per minute with horsepower HP and diameter D. The transfer to the specific con-

[5] The cavitation test stand at the Massachusetts Institute of Technology, Cambridge, shows that it is possible to study special problems in turbine design under very much simplified conditions. The phenomenon of cavitation has been demonstrated in venturi-shaped passages having two flat faces and having different angles of flare.

[6] These definitions might be given in dimensionless figures in order to be readily understood internationally, as then they are the same in metric or English units. In the author's book "Turbines and Pumps" (English translation in preparation) such definitions are given. In order to avoid complication in this paper, the terms and constants will be given only for English units.

ditions is made in two steps: First to unity head and then to unity horsepower. The changing values may be recorded in tabular form as follows:

	Head	Horse-power	Diameter	Rpm
Actual..............	H	HP	D	n
Reduce to 1 ft head.	unity	$HP \frac{1}{H^{3/2}}$	D	$n \frac{1}{H^{1/2}}$
Reduce to unit HP..	unity	unity	$D\left(\frac{H^{3/2}}{HP}\right)^{1/2}$	$\frac{n}{H^{1/2}} \frac{HP^{1/2}}{H^{3/4}} = \frac{n\, HP^{1/2}}{H^{5/4}}$

In the first step the head is reduced to 1 ft, and so the horsepower is changed to $HP \times 1/H^{3/2}$ and the revolutions per minute to $n \times 1/H^{1/2}$. In the second step the horsepower is in turn reduced from $HP/H^{3/2}$ to unity or in the ratio $1/(HP/H^{3/2})$, so that the diameter must be altered by the square root of this ratio or $1/(HP/H^{3/2})^{1/2}$, while the number of revolutions must again be changed by the inverse of this latter ratio, since ϕ must be maintained the same. Therefore $n/H^{1/2}$ times $(HP^{1/2}/H^{3/4})$ equals $n\, HP^{1/2}/H^{5/4}$.

Not only does this demonstrate that the specific speed can easily be deduced without resorting to formula, but the method of deduction lays a good foundation for the following paragraphs through exercise of the principles of transfer of head.

Now by separating the expression in Equation [34] into two parts, and inserting D outside the radical and $1/D^2$ inside, the expression becomes

$$n_s = \frac{n\,D}{H_t^{1/2}} \sqrt{\frac{HP}{D^2\, H_t^{3/2}}} \quad \text{[35]}$$

Keeping in mind that unit speed n_1, or the speed of a homologous runner at 1 ft head and 1 ft in diameter, is

$$n_1 = \frac{n\,D}{H_t^{1/2}} \quad \text{[36]}$$

and that unit power HP_1, or the power of a runner 1 ft in diameter operating at 1 ft head, is

$$HP_1 = \frac{HP}{D^2\, H_t^{3/2}} \quad \text{[37]}$$

it will be seen that Equation [35] readily resolves itself into

$$n_s = n_1\, HP_1^{1/2} \quad \text{[38]}$$

The following formulas apply equally well to pumps or turbines, the only difference being the question as to which head and power should be used, on account of the losses. The situation can be simplified by assuming for the present an efficiency of 100 per cent, so that $HP = WH/550 = Q\,\gamma\,H/550$.

Considering u as the tangential velocity of the runner at the periphery, in other words u_{peri}, it is recalled that a ratio commonly used in American practise is

$$\phi = \frac{u_{\text{peri}}}{\sqrt{2gH_t}} \quad \text{[39]}$$

For u_{peri} can be substituted $\pi n D/60$, so

$$\phi = \frac{\pi n D}{60\sqrt{2gH_t}} \text{ or } \frac{\pi}{60\sqrt{2g}} \frac{nD}{H_t^{1/2}} = 0.00652\, n_1 \quad \text{[40]}$$

Unit discharge Q_1 may be defined in a manner similar to HP_1 and n_1 as

$$Q_1 = \frac{Q}{D^2\, H_t^{1/2}} \quad \text{[41]}$$

and if the efficiency is 100 per cent

$$HP_1 = \frac{Q_1 \gamma}{550} \quad \text{[42]}$$

and hence

$$n_s = n_1\, Q_1^{1/2} \sqrt{\frac{\gamma}{550}} = 0.3368\, n_1\, Q_1^{1/2} \quad \text{[43]}$$

This equation is particularly important as forming the basis for the following, in which it will be demonstrated that specific speed can be reduced to terms of a few ratios that are characteristic of the type of velocity diagram and design of runner.

A given specific speed may be secured as the product of either a comparatively high unit speed and small unit discharge, or vice versa. High unit speed is conducive to high peripheral and relative velocities, and consequent high f iction losses. On the other hand, high unit discharge leads to large losses in the draft tube. It appears from this that if two extremes are detrimental, then there should be some optimum value of their ratio, or perhaps more than one. The problem is now:

(1) Are those ratios n_1/Q_1 which are used at present really the best ones?

(2) Which ratios should be chosen to suit the higher specific speeds that will certainly be adopted in the future?

When seeking the answer for these questions, it must be remembered that a given specific speed can be obtained by various shapes of blades, which are distinguished from each other by various combinations of $(v_{\text{tang}}\, r)_p$ and $(v_{\text{tang}}\, r)_s$, even though these moments may have the same difference. This is not true in conventional practise as long as $(v_{\text{tang}}\, r)_s$ is considered to be zero, but in a general case this limitation no longer applies, and so a further question is:

(3) Which values of $(v_{\text{tang}}\, r)_p$ and $(v_{\text{tang}}\, r)_s$ are to be selected for a given specific speed?

Although the questions are pertinent to any type of machine, it is desirable to limit this paper to a consideration of purely axial runners.[7]

Although any concentric circular section may be used, because each layer has to be designed for the same moments of whirl as well as the same difference between them, let us consider as the one most convenient to treat simply the outer one—i.e., the one at the periphery of the runner.

Now because $r_p = r_s$ for axial runners, and since $u = r\omega$, the head may be expressed

$$H_p - H_s = H_t = \frac{u}{g}\,(v_{\text{tang}\,p} - v_{\text{tang}\,s}) \quad \text{[44]}$$

We now have use for two characteristic figures in the velocity diagram, one of which has been introduced before. These are m, the ratio of the difference in tangential velocities v_{tang} divided by the tangential velocity of the runner, or u, and s, the ratio of the tangential velocity at the suction side $v_{\text{tang}\,s}$ divided by this same u. As this discussion applies to the conditions in the outer layer, u will be given the subscript $_{\text{peri}}$, to keep this fact in mind, but it should be understood that the other velocities appearing in the same equations with u_{peri} are the ones which correspond to the conditions at the periphery. Therefore

$$m = \frac{(v_{\text{tang}\,p} - v_{\text{tang}\,s})}{u_{\text{peri}}}; \qquad s = \frac{v_{\text{tang}\,s}}{u_{\text{peri}}} \quad \text{[45]}$$

[7] Professor Moody discussed the same problem in 1921, but without using the details of modern experience in aerodynamics and hydrodynamics. See Bibliography, section E, reference 5.

By substituting in Equation [44] the equivalent of ($v_{\text{tang } p} - v_{\text{tang } s}$)

$$H_t = \frac{m\, u_{\text{peri}}^2}{g} \quad \ldots\ldots\ldots\ldots [46]$$

or, because $u_{\text{peri}} = \pi n D/60$

$$H_t = \frac{\pi^2 m\, n^2 D^2}{3600 g} \quad \ldots\ldots\ldots\ldots [47]$$

or by extracting the square root of both sides and combining and setting $nD/H_t^{1/2} = n_1$

$$\frac{n\,D}{H_t^{1/2}} = \frac{60\, g^{1/2}}{\pi\, m^{1/2}} = \frac{108.4}{m^{1/2}} = n_1 \quad \ldots\ldots\ldots\ldots [48]$$

Another characteristic ratio in the design of a turbine or pump is the relation between the head corresponding to the axial velocity at the suction tip of the blades and the total head. This will be denoted by k

$$k = \frac{v_{\text{ax}}^2}{2gH_t} \quad \text{or} \quad v_{\text{ax}} = \sqrt{2gH_t k} \quad \ldots\ldots\ldots [49]$$

In case $v_{\text{tang } s}$ is zero, which means that s is also zero, k is then the per cent that the kinetic energy on the suction side of the runner bears to the total energy available. This figure is important with respect to efficiency and danger of cavitation for turbines, but for pumps it has significance only with regard to cavitation. The reason is that the suction line of a pump has accelerated flow, and as it is allowable to have the area before the runner small, a high figure of k can be used. On the other hand, the draft tube of a turbine has retarded flow, and although the passage may be enlarged very gradually, nevertheless the loss for the same value of k is greater.

Considering now that the flow may not be axial, the total kinetic energy expressed in per cent of the total head is

$$\frac{v_{\text{ax}}^2 + v_{\text{tang } s}^2}{2gH_t} = \frac{v_{\text{ax}}^2 + s^2 u_{\text{peri}}^2}{2gH_t} = k + \frac{s^2}{2m} = (2km + s^2)/2m \quad \ldots\ldots\ldots [50]$$

Now there appears still another characteristic ratio f, the net discharge area after deduction for the hub, divided by the area of the runner, $\pi D^2/4$. Then if D_h is the diameter of the hub, $f = \pi D^2/4 - \pi D_h^2/4$ divided by $\pi D^2/4$, which in turn equals $1 - (D_h/D)^2$, and so from Equation [49] it follows:

$$Q = \frac{\pi}{4} f D^2 \sqrt{2gH_t k} \quad \ldots\ldots\ldots\ldots [51]$$

from which comes the expression for unit discharge

$$Q_1 = \frac{Q}{D^2 H^{1/2}} = \frac{\pi}{4} f \sqrt{2gk} = 6.30 k^{1/2} f \quad \ldots\ldots [52]$$

Now as Equation [48] gives an expression for n_1 and Equation [51] defines Q_1, these values can now be set in Equation [43] to give

$$n_s = 0.3368\, n_1 Q_1^{1/2} = 0.3368 \frac{108.4}{m^{1/2}} \sqrt{6.30\, f\, k^{1/2}}$$

$$= 91.6 \frac{f^{1/2} k^{1/4}}{m^{1/2}} \quad \ldots\ldots [53]$$

which is most important in showing the bearing which the factors f, k, and m have on the specific speed.

First consider f. Although increase in D_h decreases the unit discharge, a larger hub is ordinarily to be preferred because of the difficulty in designing the blades near the hub if it is too small. Even though meeting these conditions by improving the design at this part of the blade is a means of securing higher unit discharge, and therefore more power in a given space, let us for the present consider f as fixed at representative average value of 0.75. In which case

$$n_s = 79.4 \frac{k^{1/4}}{m^{1/2}} \quad \ldots\ldots\ldots\ldots [54]$$

Combining the two facts expressed in Equation [45]

$$v_{\text{tang } p} + v_{\text{tang } s} = (m + 2s) u_{\text{peri}} \quad \ldots\ldots\ldots [55]$$

as will be appreciated by reference to Fig. 18. Then introducing

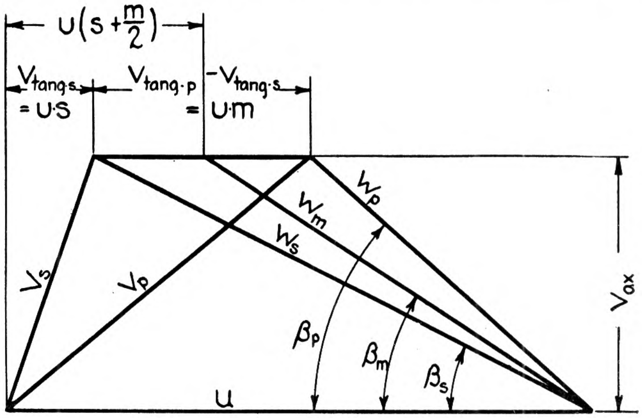


FIG. 18 GENERAL VELOCITY DIAGRAM FOR AN AXIAL RUNNER

mu for its equivalent in Equation [21] and making use of Equation [55], and removing the factor u from the parentheses, we have

$$\frac{\zeta_1 l}{2S} = \frac{m\, u_{\text{peri}}}{\sqrt{v_{\text{ax}}^2 + u_{\text{peri}}^2 (1 - m/2 - s)^2}} \quad \ldots\ldots [56]$$

which after dividing by u may also be expressed as

$$\frac{\zeta_1 l}{2S} = \frac{m}{\sqrt{(v_{\text{ax}}/u_{\text{peri}})^2 + (1 - m/2 - s)^2}} \quad \ldots\ldots [57]$$

Keeping now in mind that by definition $k = v_{\text{ax}}/2gH_t$, which transposed is $v_{\text{ax}} = k\, 2gH_t$, and also that $u^2 = gH_t/m$, the right-hand sides combined give

$$\frac{v_{\text{ax}}^2}{u_{\text{peri}}^2} = 2k\,m \quad \ldots\ldots\ldots\ldots [58]$$

Substituting in [57], this leads to

$$\frac{\zeta_1 l}{2S} = \frac{m}{\sqrt{2km + (1 - m/2 - s)^2}} \quad \ldots\ldots\ldots [59]$$

By transposing Equation [52]

$$k = \left(\frac{n_s}{79.4}\right)^4 m^2 \quad \ldots\ldots\ldots\ldots [60]$$

k may be replaced and [59] may now be rewritten

$$\frac{\zeta_1 l}{2S} = \frac{m}{\sqrt{2(n_s/79.4)^4 m^3 + (1 - m/_2 - s)^2}} \quad \ldots\ldots [61]$$

It is desirable to show graphically the meaning of these equations by a velocity diagram that is made dimensionless by simply dividing all vectors by u_{peri} as in Fig. 19. It will be seen that

$v_{\text{tang } p} - v_{\text{tang } s}$ of Fig. 18 become simply m. In the same way $v_{\text{tang } s}$ becomes s, while v_{ax} becomes $v_{\text{ax}}/u_{\text{peri}}$. By taking the square root of Equation [58], this is seen to be $(2km)^{1/2}$.

With s fixed, the locus of the left-hand end of the line m is a line perpendicular to the base line $u_{\text{peri}}/u_{\text{peri}}$ = unity. The locus of the right-hand end depends upon the specific speed, since Equation [60] may be written $k = (n_s/79.4)^4 m^2$. Consequently, for a given specific speed, k is proportional to m^2. Furthermore, as $v_{\text{ax}}/u_{\text{peri}} = (2km)^{1/2}$, it is in turn proportional to $m^{3/2}$, and so the lines of equal specific speed will have the form shown. It will be seen that this relation is independent of s, the significance of which will appear in the following section (*b*).

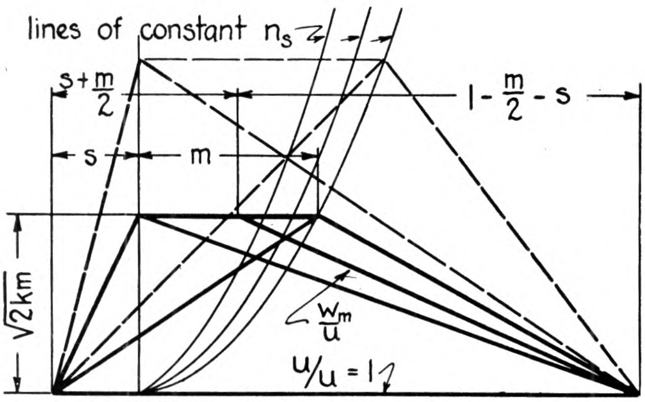


FIG. 19 DIMENSIONLESS VELOCITY DIAGRAM FOR AN AXIAL RUNNER
(Showing influence of s, m, and specific speed.)

Such a dimensionless velocity diagram is useful in showing what restrictions will be placed upon the actual diagram by a definite value of specific speed.

Equations [57], [59], and [61] also show that the three quantities determining the characteristics of the airfoil section to be selected for a blade depend also upon these ratios k, m, and s. It should be remembered that the constants used have been based upon an assumption of an average value of f, and so apply only when $f = 0.75$.

(*b*) *Consideration of a Special Profile.*

There is now the question of determining which values of m, s, and k should be selected in order to secure both high efficiency and assurance against cavitation. The method of solution is to develop and analyze equations expressed in terms of these ratios. As far as the lowest pressure, which controls cavitation, is concerned, the same treatment will apply to both turbines and pumps, but the expressions for efficiency will be seen to differ.

In the general formulas dealing with conditions conducive to cavitation, it is impossible to take into account all of the pertinent details, principally because the relations of these details to the whole have so far not definitely been determined. Furthermore, it has never been conclusively established that any one profile may be considered superior to all others with respect to cavitation. Therefore, it is necessary to consider in this discussion simply a profile which is known to have properties having a strong possibility of satisfying the requirements for good pressure distribution. It is possible that such a method of attack will not give very good efficiency for the first trial, but at least the results will be useful in showing the general relations and the underlying laws, and so serve as a foundation for further search for perfection.

The profile proposed for consideration has a crescent shape. It is formed as shown in Fig. 20 by the arcs of two eccentric circles, and may be defined by three principal dimensions: l, as before, the length between two perpendiculars to the chord, but as we do not consider at first any rounding of the edges, l is actually the length of the chord itself; t, the thickness; and a, the altitude of a segment formed by the chord and a circle passing through the middle of t at the center of the blade. As a and l define the radius of curvature, the ratio a/l may be considered as a measure of the curvature of the blade, and in like manner the ratio t/l may be called the relative thickness.

The property of this profile which is particularly interesting is that if it is exposed to flow parallel to the chord, so-called "shockless approach," the lowest pressure will occur at the center of the blade, and the pressure distribution will be similar to that shown in Fig. 5, which was contrasted with Fig. 4 to demonstrate that the maximum value of Δp for a given average can be smaller if the distribution is even. The pressure along the blade is given for three examples in the lower part of Fig. 20. One of these is contrasted with the pressure distribution of an actual airfoil having the same total lift, and the superiority of the crescent in this respect is readily appreciated. This is particularly desirable, for if the reduction of pressure is not only unnecessarily great, but also occurs near the leading tip, then in large propeller turbines the conditions are aggravated by the fact that this point of lowest pressure is by the very size of the runner raised considerably above the center line and so is at a point where the static pressure is already low.

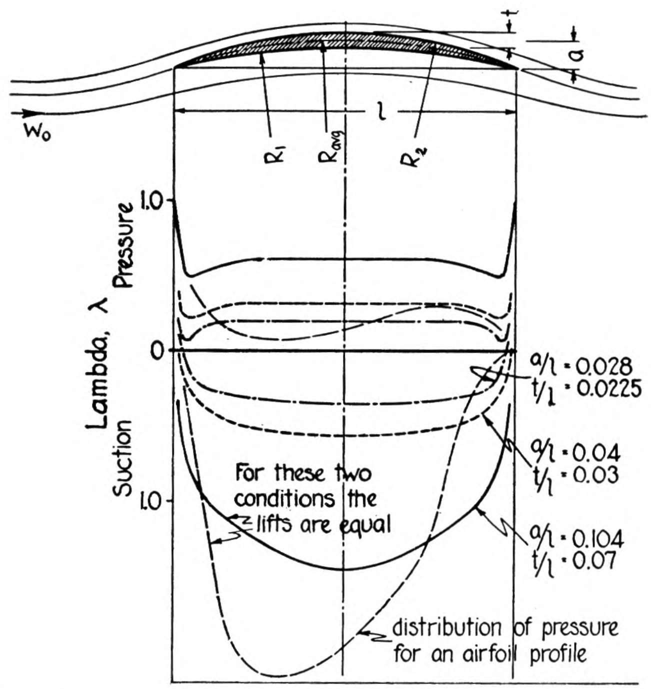


FIG. 20 CRESCENT-SHAPED PROFILE AND DIAGRAM OF PRESSURE DISTRIBUTION

Such a profile is readily adaptable to theoretical computation of pressure distribution, which has later been verified by experiment, and it has been found that the lift coefficient ζ_l in Equation [10] becomes $4\pi a/l$ for shockless approach. Substituting then in the expression $\zeta_l l b \gamma w_0^2/2g$ and setting $b = 1$, we have for lift per unit of breadth

$$L = 4\pi\, a\, \gamma\, v_0^2/2g \qquad [62]$$

It is very interesting to note that in this particular case the l's cancel, and so the lift becomes independent of the length of the chord.

In Fig. 20 the pressures have been given in terms of λ, as is customary, as λ is useful in expressing the ratio between reduction or increase in pressure at various points and the pressure corresponding to the velocity of the approaching flow. For example, the pressure at the nose is always the stagnation pressure or exactly $+1$. The λ for any point located at a distance of x from the trailing edge will be termed λ_x. Furthermore, if the shape of this curve is the same regardless of the velocity of approach, that is to say if the pressure at any point can always be expressed as a constant times the velocity pressure of the approaching flow as long as the angle of attack is the same, then it follows that the ratio between λ_x and the lift coefficient ζ_l is constant and may be designated by μ.

$$\mu = \lambda_x/\zeta_l \qquad [63]$$

The lowest value of λ, which may be denoted as $\lambda_{\min}$, occurs for this profile at the mid-point, and is of particular interest. Fig. 21 shows its value as computed for various degrees of curva-

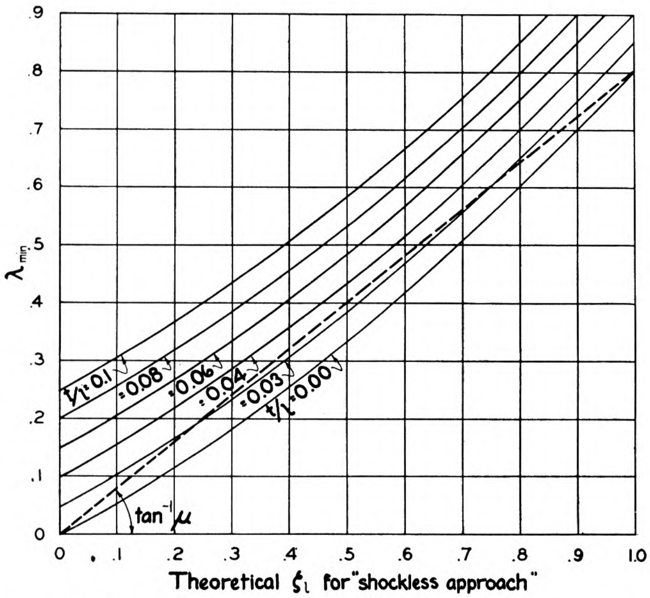


FIG. 21 LAMBDA AS A FUNCTION OF LIFT COEFFICIENT FOR SHOCKLESS APPROACH AND FOR VARIOUS THICKNESSES OF BLADE

ture and for various thicknesses of blade. It will be seen that $\lambda_{\min}$ increases practically in proportion to ζ_l, and that it is also affected by increasing thickness of the blade. Leaving strength out of consideration for the present, it is apparent that the average relationship between $\lambda_{\min}$ and ζ_l may be represented approximately by a straight line having a slope, which is the μ referred to above, of 0.8. Consequently, for this profile $\lambda_{\min}$ becomes $3.2\pi a/l$, which multiplied by $\gamma w_m^2/2g$ gives the minimum pressure on the blade with respect to the surrounding pressure, w_m now corresponding to the approach velocity v_0.

$$p_{\min} = \lambda_{\min}\frac{w_m^2}{2g} = \mu\,\zeta_l\,\frac{w_m^2}{2g} = 3.2\pi\,\frac{a\gamma}{l}\,\frac{w_m^2}{2g} \qquad [64]$$

(c) *Effect of m, s, and k on Cavitation.*

Having determined the properties of the special profile, it now remains to express the lowest pressure in terms of the ratios m, s, and k, for the assumed values for ϵSl and μ. Only an outline of the mathematics will be given on account of the lack of space.

Inasmuch as λ_x for any given point, multiplied by the square of the mean relative velocity, gives the depression below the mean pressure p_m/γ, then

$$\frac{\lambda_x w_m^2}{2g} = \frac{p_m}{\gamma} - \frac{p_{\min}}{\gamma} \qquad [65]$$

for p_m/γ may be written $p_s/\gamma + (w_s^2 - w_m^2)/2g - x$, x being as before the vertical height from the suction side of the runner to the point of lowest pressure.

It also is true that the pressure at the suction edge of the blades is equal to the barometric height H_b, minus the draft head, or lift, H_s, measured from the free water surface to the lower tip of the blade, again minus an expression which is equal to the efficiency of the draft tube ϵ_s times the velocity head at that point. Thus

$$\frac{p_s}{\gamma} = H_b - H_s - \epsilon_s\,\frac{v_{\text{ax}}^2 + v_{\text{tang}\,s}^2}{2g} \qquad [66]$$

and so by substitution and transposition, and because under the conditions where cavitation is just beginning, the lowest pressure is equal to the vapor pressure H_v, we get

$$H_b - H_v - H_s - x = \epsilon_s\,\frac{v_{\text{ax}}^2 + v_{\text{tang}\,s}^2}{2g} - \frac{w_s^2 - w_m^2}{2g} + \frac{\lambda_x\,w_m^2}{2g} \qquad [67]$$

This Equation [67] has been written so that the terms $H_p - H_v - H_s - x$, $H_s + x$ together being the draft head to the point of lowest pressure and commonly written just H_s, the point of reference usually being taken arbitrarily, stand on the left-hand side, so that when divided by $\Delta^p{}_sH$, commonly called just H, they will equal the familiar expression for Σ. Dividing the other side of the equation by the equivalent of $\Delta^p{}_sH$, which is equal to mu_{peri}^2/g, we have

$$\frac{H_b - H_v - H_s - x}{\Delta_s{}^pH} = \Sigma$$

$$= \frac{\epsilon_s(v_{\text{ax}}^2 + v_{\text{tang}\,s}^2) - (w_s^2 - w_m^2) + \lambda_x\,w_m^2}{2m\,u_{\text{peri}}^2} \qquad [68]$$

Because the losses that occur in a turbine or pump have not been taken into account, which means that $\Delta_s{}^pH$ has to be used instead of the total head H_t as is customary in determining Σ, and also because ideal conditions were assumed, such as sharp leading edges etc., it cannot be expected that the values that would be computed from Equation [68] and those which are later derived from it, would agree exactly with actual cavitation tests. Nevertheless, such formulas are useful in showing general relationships and the effects of variables upon Σ.

It is necessary to free Equation [68] from the terms v_{ax}, $v_{\text{tang}\,s}$, w_s, w_m, and λ_x. $w_s^2 - w_m^2$ may first be eliminated by consideration of Fig. 18, whereby they may be reduced to the expression

$$w_s^2 - w_m^2 = \left[2(1 - s) - \frac{m}{2}\right]\frac{m}{2}\,u_{\text{peri}}^2 \qquad [69]$$

By substituting $2km$ for $v_{\text{ax}}^2/u_{\text{peri}}^2$ and considering that $v_{\text{tang}\,s}^2/u_{\text{peri}}^2$ is equal to s^2, then

$$v_{\text{ax}}^2 + v_{\text{tang}\,s}^2 = (2km + s^2)u_{\text{peri}}^2 \qquad [70]$$

Referring to Equation [56], we find that ζ_l is equal to $\dfrac{2S}{l}\,m\,u_{\text{peri}}$

divided by an expression which is equivalent to w_m. It follows that

$$\lambda_x = \mu \zeta_l = \frac{2\mu}{l/S} \frac{m\, u_{\text{peri}}}{w_m} \quad [71]$$

Now finally

$$w_m = u\sqrt{2km + (1 - m/2 - s)^2} \quad [72]$$

and so by these substitutions the basic formula is derived

$$\Sigma = \frac{m}{8} + \epsilon_s k + \frac{1}{2}\left(s + \frac{\epsilon_s s^2}{m}\right) + \frac{\mu}{l/S}\sqrt{2km + (1 - m/2 - s)^2} - \frac{1}{2} \quad [73]$$

It will be noted that there appear constants which must be considered as given: ϵ_s = efficiency of draft tube, which is not the same for pumps as for turbines; S = spacing between blades; l = the length of the blade; μ (as explained in preceding section) = the ratio between λ_x and ζ_l, as well as the three ratios m, s, and k.

Σ has been computed for conditions which will be assumed for the sake of demonstrating the effect of the ratios m, s, and k, not only upon cavitation but upon efficiency as well (see following section (*d*)).

Three specific speeds have been selected, 125, 175, and 225. Inasmuch as it is well known from experience that with the higher specific speeds it is necessary to incline toward narrower blades, two ratios of l/S have been taken for each speed. Because these are not the same for each, no final curves plotted as a function of specific speed will be shown for fear that they would give the impression that only one variable had been changed. These assumptions are:

Specific speed n	Wider blades l/S	Narrower blades l/S
125	1.0	0.5
175	0.9	0.45
225	0.8	0.4

Fig. 22 shows for n_s = 175 the values of Σ plotted against s for constant m and gives the picture for turbines and pumps with both wider and narrower blades. It should be noted that as Σ decreases with lower values of m, s does not change greatly. It is furthermore interesting to see that Σ continues to decrease with diminishing m, so that there is no optimum value of m as far as cavitation is concerned. The same was found to be true with both the higher and the lower specific speeds, and so it is impossible to show the effect of changing speed for best conditions of cavitation. The effect will be shown later when the best m for efficiency has been determined.

Comparison of the four panels shows plainly that narrower blades are conducive to cavitation and that conditions for pumps are more severe than for turbines.

(*d*) *Influence of m, s, and k on Efficiency.*

As previously mentioned, the question of efficiency for pumps and turbines must be treated separately. Therefore, distinct sets of equations will have to be developed to suit these two cases. In general, however, it may be said that the total losses consist of three parts: (1) the loss in the suction line or draft tube, (2) the loss in the runner itself, and (3) the loss in the diffuser, or in the scroll case and distributer. Inasmuch as head loss has previously been designated by H_f, these three components will be given appropriate subscripts, H_{fs}, H_{fr}, and H_{fp}, respectively.

In computing H_{fs}, the difference between pumps and turbines is based upon the fact that in the former the flow is accelerated, whereas in the latter the kinetic energy must be converted into pressure. This process is more difficult and the loss is always greater.

For pumps, the loss at entrance under favorable conditions could not be expected to exceed that of the good nozzle, which is but a few per cent of the velocity head, $v_s^2/2g$. Thus H_{fs} may be expressed in terms of a constant c_s times the square of the axial and tangential components $c_s\ (v_{\text{ax}}^2 + v_{\text{tang}\ s}^2)/2g$. For $v_{\text{tang}\ s}^2$ may be written $u^2 s^2$, whereas for v_{ax}^2 may be written $2kmu^2$, so

$$H_{fs} = c_s\, u_{\text{peri}}^2(2km + s^2)/2g \quad [74]$$

In the case of turbines, it is only necessary to substitute in place of c_s in Equation [74] the expression $1 - \epsilon_s$, ϵ_s being draft-tube efficiency referred to in the section on cavitation. Thus

$$H_{fs} = (1 - \epsilon_s)u_{\text{peri}}^2(2km + s^2)/2g \quad [75]$$

It may be a question whether ϵ_s may be assumed to be constant for all values of whirl at the outlet of the runner. This certainly would not be true for any tube selected at random, but experience has shown that at least for small values of s the draft

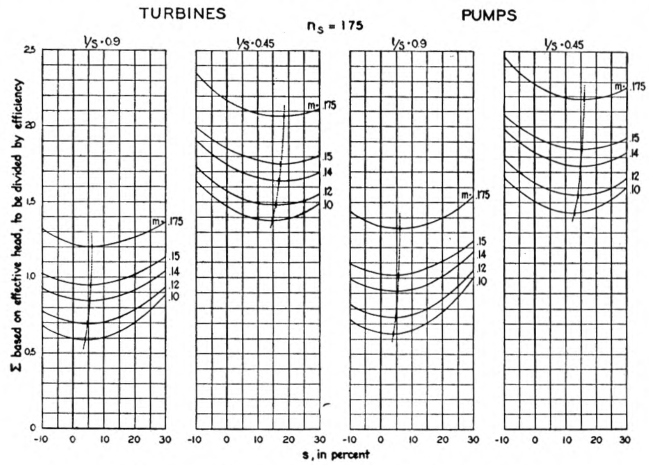


FIG. 22 VALUES OF Σ PLOTTED AGAINST s

tube efficiency can be the same as, or even a little better than, that for a straight axial flow. The author in his laboratory at Karlsruhe, Germany, has had experience with a special form of tube used with a Kaplan runner and investigated by Dr. Krisam.(12) This tube had an extreme amount of flare, being much shorter and more spreading than those proposed by Moody, and was especially shaped to avoid the amount of excavation required for an elbow tube. The results compared very favorably with the straight conical tube that was selected out of a series of five as being the most suitable for this particular wheel. It showed better efficiency than the elbow tube, and in extreme cases of whirl it was even better than the straight conical tube.

On the basis of this experience it may be assumed that if the turbine is not too small, ϵ_s may be considered to be a constant and to have a value of 0.875.

Consider now the losses in the runner itself which are caused essentially by the work absorbed in overcoming the drag of the blade passing relative to the water with a velocity of w_m. The work lost in the incremental layer at the periphery is therefore dDw_m and must be supplied by the partial discharge passing through this incremental layer in the space between a single pair of blades. This discharge is therefore $v_{\text{ax}}\ \gamma\ S\ dr$, and the head loss is H_{fr}. Consequently,

$$dD\, w_m = \gamma\, v_{\text{ax}}\, S dr\, H_{fr} \quad [76]$$

Recalling Equation [13] for dD, which multiplied by w_m gives

$$dD\, w_m = \zeta_d\, l\, dr\, \gamma\, w_m^3/2g = \gamma\, v_{ax}\, S\, dr\, H_{fr} \ldots\ldots [77]$$

canceling γ and dr, and by setting w_m^3 equal $u^3\ [2km + (1 - \frac{m}{2} - s)^2]^{3/2}$ and v_{ax} equal $u\sqrt{2km}$, then canceling u, H_{fr} may be expressed

$$H_{fr} = \frac{\zeta_d}{2g}\,\frac{l}{S}\,\frac{u_{peri}^2\,[2km + (1 - m/2 - s)^2]^{3/2}}{\sqrt{2km}} \ldots\ldots [78]$$

This applies for turbines or pumps, except that for pumps ζ_d must be assumed to be slightly higher than for turbines. Fair values may be considered approximately 0.015 and 0.010, respectively.

On the pressure side the losses again differ for a reason which corresponds with that given for the suction side, but reversed.

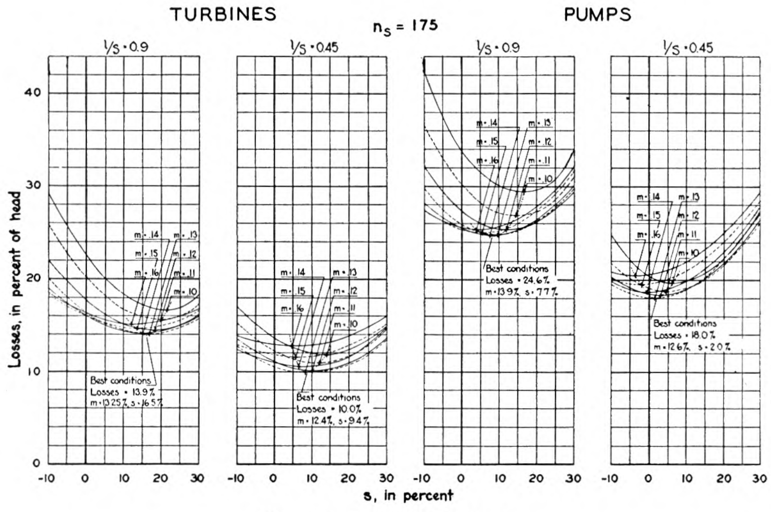


FIG. 23 LOSSES VERSUS s

The same design of water passages that are used for turbine draft tubes might be used for pumps except that commercially it would not be possible to sell them with the size of diffuser that would be required. There is less incentive in this direction in the case of higher head pumps with longer pipe lines because the high velocity of discharge may be used to save in the diameter of the pipe line. In such a case it is not fair to charge against the pump the entire loss corresponding to the high velocity of discharge, but for the purpose of this paper, which deals primarily with low-head design, it will be considered that the discharge line is short and that it would be desirable to reconvert as much kinetic energy as possible into head. Consequently, it will be assumed that the efficiency of the diffuser is 0.80.

From Fig. 18 it will be seen that the discharge velocity squared v_p^2 is equal to $v_{ax}^2 + (su_{peri} + mu_{peri})^2$. Multiplying the entire expression by u_{peri}^2 and dividing each term individually by the same, and then replacing by their equivalent in terms of s, m, and k, gives

$$H_{fp} = \frac{(1 - \epsilon_p)}{2g}\, u_{peri}^2\ [2km + (s + m)^2] \ldots\ldots [79]$$

For turbines the expression is similar except that in place of $1 - \epsilon_p$ a coefficient corresponding to that used for the suction line of pumps, c_p, must be used. Thus

$$H_{fp} = \frac{c_p}{2g}\, u_{peri}^2\, [2km + (s + m)^2] \ldots\ldots\ldots [80]$$

Recapitulating, formulas [74], [78], and [79] may be combined and divided by $\Delta_s{}^pH$ on one side and by its equivalent $m\, u_{peri}^2/g$ on the other side, and then written in the form

$$\frac{H_f}{\Delta_s{}^pH} = \frac{c_s(2km + s^2)}{2m} + \frac{\zeta_d}{2m}\,\frac{l}{S}\,\frac{[2km + (1 - m/2 - s)^2]^{3/2}}{\sqrt{2km}} + \frac{1 - \epsilon_p}{2m}\,[2km + (s + m)^2] \ldots\ldots [81]$$

In like manner Equations [75], [78], and [80] may be combined and written in the form

$$\frac{H_f}{\Delta_s{}^pH} = \frac{(1 - \epsilon_s)(2km + s^2)}{2m} + \frac{\zeta_d}{2m}\,\frac{l}{S}\,\frac{[2km + (1 - m/2 - s)^2]^{3/2}}{\sqrt{2km}} + \frac{c_p}{2m}\,[2km + (s + m)^2] \ldots\ldots [82]$$

These formulas give the losses in per cent of the head effectively used so that appropriate adjustment must be made to express these losses in per cent of the total head H_t. Remembering now that for a given specific speed, k may be replaced by $(n_s/79.4)^4m^2$, and so given the n_s and l/S, the losses may be computed for various combinations of m and s. This has been done for three specific speeds using the same blade width l/S as in the previous section on cavitation.

The results for specific speed of 175 are presented in Fig. 23, which shows the losses for turbines and pumps having l/S of both 0.9 and 0.45. It will be noted that all of these curves have a minimum for some s, and that for a combination of m and s the best conditions, as far as efficiency is concerned, are found as illustrated by the minimum of the dotted curve passing through the minima of the individual curves. The same was done for specific speeds of 125 and 225, the results being shown in Table 1.

TABLE 1 VALUES FOR MINIMUM LOSSES (IN PERCENTAGES)

n_s	l/S	Turbines Losses	Turbines m	Turbines s	Pumps Losses	Pumps m	Pumps s
125	1.0	9.2	20.0	16.4	16.6	23.6	0.4
	0.5	6.8	17.5	13.5	12.8	21.0	−4.6
175	0.9	13.9	13.2	16.5	24.6	13.9	7.7
	0.45	10.0	12.4	9.4	18.0	12.6	2.0
225	0.8	19.2	9.5	15.0	31.8	9.8	8.6
	0.4	14.2	8.5	10.0	23.5	8.7	4.5

It should be noted that for higher specific speeds there is a tendency for the losses to increase in spite of the beneficial effect of the narrower blades. The best value of m decreases with increase in n_s, whereas the whirl s is inclined to increase with pumps, but shows no marked tendency for turbines. Furthermore, it is interesting to see that for the lower specific speeds and narrower blades there is an advantage in having the flow approach the pump with a negative whirl.

It must be remembered that when the crescent profile was proposed it was not expected that good efficiency could be secured at the first trial, and so it should not be assumed from the table that the values for losses represent by any means the best that can be expected. This table is simply to show the general tendencies. Furthermore the curves do not represent the performance that any one runner will give, as it is necessary to alter the design to correspond with the changes in the velocity diagrams.

(e) *General Discussion and Conclusions.*

Returning now to Fig. 23, comparison between the four panels shows plainly that in general the losses in the case of a pump are greater than those for a turbine. As explained before, this is largely because adequate diffusers cannot be provided at reasonable cost. It is also plain that in both turbines and pumps the narrower blades are more conducive to higher efficiency. This should now be contrasted with Fig. 22, which shows the values of Σ for these same conditions. In this respect wider blades are more advantageous, since the narrower blades are more conducive to cavitation. Consequently, it is necessary to strike a compromise sacrificing to some extent efficiency for the sake of better Σ.

In order to give a better idea how Figs. 22 and 23 may be coordinated, the corresponding values of losses and Σ have been listed for various combinations of m and s, the data being taken for the wider bladed turbines. Curves for constant m showing Σ plotted against losses are shown in Fig. 24. It will be seen from this diagram that if the designer could assign a comparative value between depth of excavation required to satisfy Σ and the capitalized value of loss in efficiency, it would be possible to arrive at the economical solution in the manner indicated, and so determine the proper m and s to suit the conditions. It would be necessary then to investigate for various ratios of l/S to be sure that all variables are properly balanced to give the best results.

Now that the results showing the proper combination of m and s are approximately known, it is interesting to see the effect of specific speed upon Σ. In preparing Fig. 25 the m that would probably be selected as a compromise between cavitation and efficiency has been bracketed by two values between which the most economical may be considered to lie. This was done for each specific speed, and examination of the diagram shows plainly that with higher speed the value of Σ increases very materially, a fact which adds greatly to the cost of setting. It is interesting to note that at the same time the design becomes more sensitive to having the proper component of whirl, and although the broader blades have their best value of s at approximately 5

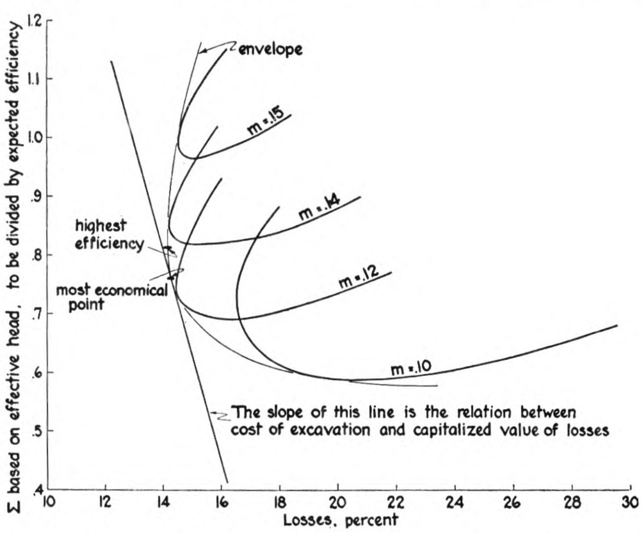


FIG. 24 COMPROMISE IN SELECTION OF m BETWEEN EFFICIENCY AND CAVITATION

per cent, the narrower blades require that the whirl should be increased.

In the interpretation of these curves it should be borne in mind that they have been worked out for one profile for which the ratio λ_{min} to lift coefficient ζ_l is 0.8. These calculations may be applied to another profile providing this ratio is substantially constant. Equation [73] shows that if μ can be made smaller, Σ will be affected favorably. Therefore, there is a distinct advantage in finding a profile which will have a very even distribution of pressure similar to that shown in Fig. 5. The foregoing suggests that the solution may be in finding a blade shape that will have a constant or nearly constant rate of change of whirl component.

Although these curves have been worked out for a special profile, it should be considered that as long as the distribution of pressure in terms of $V_0^2/2g$ is independent of the approach velocity itself, then there is a constant ratio μ between the pressure

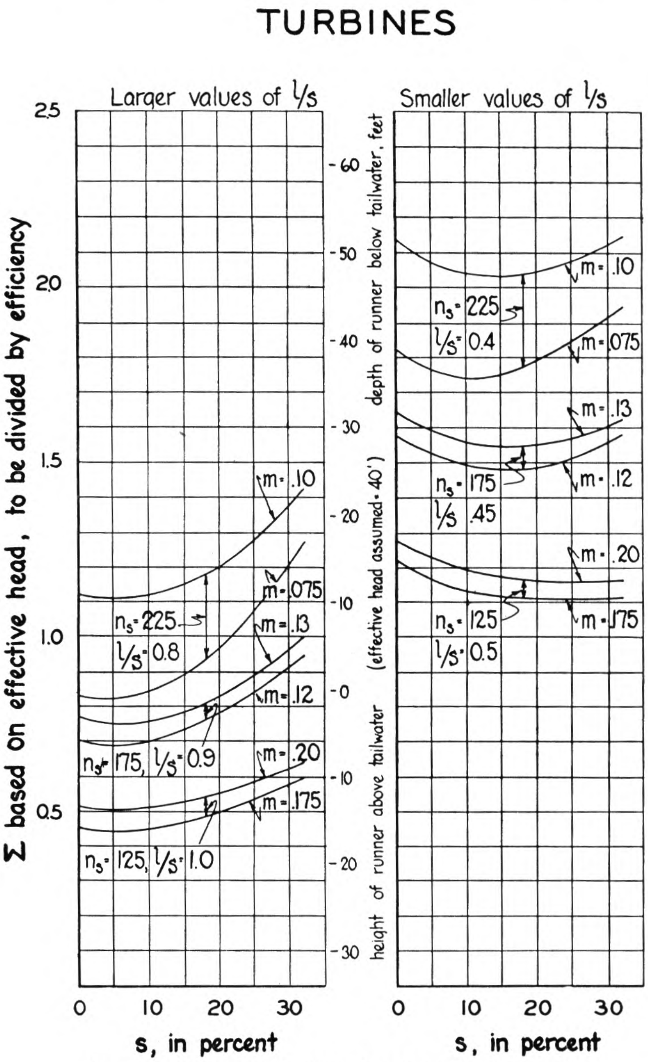


FIG. 25 EFFECT OF SPECIFIC SPEED ON CAVITATION

at any point and the lift coefficient. Consequently, as long as the angle of attack α is maintained by the alteration of the position of the blade to suit the changing angle β_m, then the method outlined is general and might apply to any profile.

As a further refinement, it must not be overlooked that the point of actual lowest pressure on a runner, particularly one with large dimensions, may not be identical with that corresponding to λ_{min} because pressure is decreased with increase in the distance x above the lower tip of the blade. Consequently, it may be necessary to determine the pressure of points slightly above the one where the lowest pressure would otherwise be expected.[8]

[8] Investigations are at present under way at Karlsruhe to determine the effect of the shift of the point of lowest pressure with change of scale. This shift, the amount of which depends upon the characteristics of pressure distribution, is one of the principal uncertainties in transferring cavitation results from models to prototypes.

While making calculations for the results shown in Figs. 22 to 25, the effect of friction upon s was very impressive. It becomes evident that when friction is but a small part of the losses, there is little interest in increasing s, but when friction is high, it is better to sacrifice loss from increased kinetic energy at the suction side of the runner in order to reduce the relative velocity which affects the friction loss in proportion to the square. This opinion was expressed by the author in his lecture before the Hydraulic Power Committee of the National Electric Light Association, February, 1932,(13) although at that time quantitative results were not shown. The value of whirl in the draft tube was realized previously by Moody.(14)

From the fact that high friction tends toward high values of s, it may be deduced that the presence of high whirl when a unit is operating at point of best efficiency is an indication of high friction and possibly defective blade profile.

It would be interesting to continue these mathematical investigations to analyze the conditions in the inner layers of the turbine, but to do so is beyond the scope of this paper. It is hoped that the studies that have been made so far will prove useful in emphasizing the underlying principles that must be observed if successful design is to be secured by means other than by trial and error, an evolutionary process that is costly and time consuming.

In conclusion the author would like to leave the impression that the attractive advantages of higher specific speed, which open the possibility for hydro development in fields that otherwise would be closed, are going to create a demand for further achievement. It will be appreciated, however, that the prize is not easily won, for with higher speeds the design is exceedingly sensitive to having the proper conditions in every respect. To realize these advantages it is therefore very necessary that we should seek a better knowledge of the problem and solve one by one the many unknown factors that have been enumerated in this paper.

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Numbers in parentheses appearing in text refer to bibliography as follows:

	See reference below		See reference below
1	E-1	8	A
2	A-4,5,6,7,8	9	E-2
3	A-1,2,3,7,8,9	10	B-1
4	B	11	E-4
5	C	12	D-2
6	C, E-2	13	N.E.L.A. 222
7	B, C	14	E-5

Abbreviations: *Z.A.M.M.*, *Zeitschrift für angewandte Mathematik und Mechanik*, published by von Mises, Berlin. *Mitteil. Institut*, *Mitteilungen aus dem Institut für Strömungsmachinen der Technischen Hochschule Karlsruhe*. *V.D.I. Zeit.*, *Zeitschrift des Vereines deutscher Ingenieure.*

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NOTE: This bibliography is necessarily incomplete, and drawn from sources nearest at hand. For a more complete list of references, see Engineering Index.

Symbols

For convenience, the symbols used in this paper have been listed in alphabetical order.

A = area, sq ft
A_p = projected area, sq ft
a = altitude of segment formed by chord and mean circle of crescent profile, ft
b = breadth, ft
c = coefficient of discharge applied to intake of pump and distributor of turbine
cg = center of gravity
D = drag, lb
D = (also) diameter of runner, ft
D_h = diameter of hub, ft
d = sign of differential
F = force, lb
f = ratio $D^2 - D_h^2/D^2$
g = acceleration of gravity, 32.2 ft per sec^2
H = specific energy per unit of weight, ft
H_f = head lost through friction, etc., ft
H_{fs} = head lost at suction, ft
H_{fr} = head lost in runner, ft
H_{fp} = head lost in diffuser or scroll case, ft
HP = horsepower, ft-lb per sec ÷ 550
H_t = $H_2 - H_1$, total head, ft
$\Delta_1{}^2H$ = net or effective head, ft
h = elevation of point of pressure measurement, feet above datum plane
k = ratio between velocity head at suction side of runner and effective head
L = lift, lb
l = length of blade between two perpendiculars to the chord, ft
m = ratio between change of tangential velocity and velocity of runner
n = number of revolutions per minute
n_s = specific speed, rpm
p = pressure, lb per sq ft
Δp = differential pressure, lb per sq ft
$p_{\min}$ = minimum pressure, lb per sq ft
Q = quantity of water, cfs
r = radius, ft
r_{cg} = radius center of gravity to center of shaft, ft
S = spacing, or pitch of runner blades, ft
s = ratio tangential velocity at suction to speed of runner
T = torque, lb at 1 ft radius
t = thickness of blade, ft
u = tangential velocity of a point on the runner, ft per sec
v = absolute velocity, ft per sec
v_0 = velocity of approach to fixed profile, ft per sec
W = quantity of water, lb per sec
w = relative velocity, ft per sec
X = height of runner from suction to pressure side measured perpendicularly to direction of motion
x = distance measured along X from suction side, ft
z = number of blades
α = angle of attack, deg
β = angle formed by flow approaching or leaving the runner with reference to direction of motion, deg
γ = density, or specific weight for water, 62.4 lb per cu ft
Δ = difference
ϵ = efficiency, per cent
ζ_d = drag coefficient
ζ_l = lift coefficient
θ = angle formed by chord of blade to direction of motion
λ = ratio pressure at any point on the blade to velocity pressure corresponding to v_0
μ = ratio λ/ζ_l
Σ = cavitation coefficient
ϕ = ratio of velocity of point on periphery of runner to spouting velocity corresponding to total head
ω = angular velocity, cu ft per sec

Discussion

ARNOLD PFAU.[9] Much was published shortly before the outbreak and after the stagnation caused by the World War about turbines of high specific speeds. Theories were advanced which, on practical application, failed to bring about the expected and desired results. Again results obtained in a rather empirical manner were explained by theories of which one could not but gain the impression that the theory was made to fit the results, rather than to produce results.

The actual results obtained by Prof. Dr. V. Kaplan abroad and simultaneously to some extent by Forest Nagler in our country were gratifying and certainly most valuable for practical application, yet the writer personally was not satisfied to accept them without having at least in a simple, practical way an explanation of what is actually taking place in a propeller runner.

Prof. Dr. Hans Baudisch, of Vienna, published in 1914 his theory of the so-called suction-jet turbine, which appealed to the writer as having real merit. Unfortunately, a runner designed purely along his principles did not perform correctly, because it neglected other more practical factors, the effect of which was fundamental.

In Prof. Dr. Baudisch's axial-discharge suction-jet runner he attempted to compel the water to decelerate in its relative flow through the runner channels. These flared too much, because too short; the water could not decelerate as fast, with the result that it cut loose from the walls of the channels; or in other words, it did not keep the channels filled, so that the desired deceleration and subsequent suction effect did not materialize. The old relation:

$$W_2^2 - W_1^2 = C_\rho^2 < 0$$

defining the so-called reaction velocity, or reaction pressure

$$p_\rho = \frac{C_\rho^2 \gamma}{2g}$$

did not apply, because W_2 did not become smaller than W_1. No negative pressure p_ρ was produced, so that neither speed nor discharge capacity came up to expectation.

In a propeller runner, and particularly in one of a projected blade area smaller than the full circle area, we cannot deal with actually existing runner channels, because the lower end of one blade does not overlap the upper end of the preceding blade. Consequently, we cannot speak of a relative discharge angle in the sense in which it would permit the use of our old customary velocity diagrams. It is here that a pictorial explanation assisted the writer in obtaining a clearer conception.

How does the water flow, and in what manner does it leave the propeller runner? At the time of his studies in 1916, there was no means of obtaining the stroboscopic pictures which are now available. The writer's studies convinced him, however, that the most vital process takes place "not within but below" a propeller runner. This at once pointed to the fundamental difficulties to be encountered in the use of a propeller turbine. It deals with negative pressures, which naturally are limited to the barometric value, whereas overpressure, in case of the reaction turbine, and atmospheric pressures, in case of the impulse wheel, never become negative, no matter how high the head may be.

Again using the old relation:

$$W_2^2 - W_1^2 = C_\rho^2, \text{ or } a\, p_\rho$$

we have:

Reaction turbines: $W_2^2 - W_1^2 = C_\rho^2 > 0;\ p_\rho > 0$

Impulse wheel: $W_2^2 = W_1^2;\ C_\rho = 0;\ p_\rho = 0$

Propeller turbines: $W_2^2 - W_1^2 = C_\rho^2 < 0;\ p_\rho < 0$

Taking the values of the coefficients of the diagram

$$W_2^x = \frac{W_2}{\sqrt{2gH}}; \qquad W_1^x = \frac{W_1}{\sqrt{2gH}}, \text{ etc.}$$

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we find at once:

$$2gH(W_2^{x2} - W_1^{x2}) = 2gHC_p^{x2} \equiv b\,p_p^x\,H$$

from which we conclude that in the case of the propeller (or the suction-jet turbine directly applicable to Professor Baudisch's theory) the value

$$\sigma\,p_p^x\,H < 0$$

can attain, or even would exceed the barometric limit B.

This immediately points out the danger of failure of performance of a propeller turbine, when used to operate under a too high head or a too high elevation above sea level (B'), or by reason of design of a too great hydrodynamic suction effect p_p, aside from the effect of the draft tube.

It may be permitted to state that it was the realization of these effects which prevented the application of the propeller in cases where failure of satisfactory performance would have been certain.

In the writer's paper, entitled "Permissible Suction Head of High-Speed Propeller Runners," A.S.M.E. Trans., HYD-51-9, he has fully explained what in his opinion actually takes place

FIG. 26

underneath a propeller runner. Fig. 26 is a reproduction from the cover of the March-June, 1931, bulletin of Escher-Wyss & Co., Zurich, Switzerland, and plainly discloses the "hypothetical channel," with its contraction and subsequent expansion, resulting in a deceleration of the relative water velocity and of the "surface of discontinuity." The irregularity of this surface as evidenced is caused by the interference of the angle at the top of the propeller blades and the actual stream-flow lines of the approaching water, as also by the shape of the rear surface of the blade, and it clearly supports the necessity of adjusting also the runner blades in order to maintain a most favorable efficiency throughout, all as introduced by the design of Prof. Dr. V. Kaplan's adjustable-blade propeller.

The more nearly the stream-flow lines correspond with the angle and the more the rear surface of the blade conforms to the conditions of a correct diffuser, the more perfect will be the "hydrodynamic suction" effect p_p; the smoother also will be the surface of discontinuity and the farther removed will be the danger of cavitation.

W. WATTERS PAGON.[10] The paper gives a clear and comprehensive discussion of the general principles of turbine and pump design and the problems awaiting solution in this field. The suggested use of higher specific speed as a result of better understanding of flow phenomena is interesting.

In aerodynamics there has been accumulated a vast amount of knowledge and data that should help greatly in the hydraulic field. The boundary layer, turbulence, form drag, viscous flows, and other general questions are coming to be understood much better. The aerodynamic research worker has gone back to Osborne Reynolds' methods and followed new paths. The problem of similitude between model and prototype involves similarity of geometric flow as well as of body, and the character of flow has been found to vary systematically with Reynolds' number, but also with such factors as turbulence, roughness, eddy formation, etc. As the author's work continues, it is the writer's belief that he will come in time to consideration of the theory of discontinuous flow. The author's work in his laboratory at Karlsruhe has already produced valuable results.

The writer has found evidence of the effects of turbulence and what in aerodynamics is called "burble," and apparent evidence of radial flow which may be associated with the varying circulation along the blade which in airplanes causes "induced drag."

The writer hopes that this paper will stimulate research and experiment looking toward the elimination of cavitation and the improvement of efficiency.

R. E. B. SHARP.[11] The writer believes that this work will stand as a most important advance in rationalizing the design of turbine and pump runners of high specific speed and that it deserves careful study. A valuable feature is the consideration of both the practical and theoretical aspects. The writer would make the following comments:

As to sections 7 and 8, the writer agrees with the importance of establishing the best form of blade for a given layer or section through the runner, in order to set up as uniform a value of pressure distribution ΔP as possible, in this way establishing the correct form of the runner blade at this section.

Whether the correctness of these sections, as demonstrated by tested values of ΔP, obtained from stationary blades, can be thus established as holding true also for revolving blades, as in an actual runner, would appear to be somewhat open to question, although this method of analysis is undoubtedly of considerable value in view of the impossibility of determining this pressure distribution in an actual runner, as pointed out by the author.

The author states that, among other incidental problems, is the influence of the design and location of the guide vanes on the manner in which the whirl moment $v_{\text{tang}}\,r$ varies across the space between the runner periphery and the runner hub. The writer is of the opinion that the manner in which this whirl component is distributed is of great importance as affecting decidedly the design of the runner blades across this space, in order to deal properly with the true values of the whirl moment. Not only is it important to consider the design and location of the guide vanes actually to be used, as affecting the whirl component on the pressure side of the runner, but it is also equally important to know the characteristic of the draft tube to be used, as affecting the manner in which the whirl moments at the runner discharge vary across the radial blade space. Unless some definite information about the foregoing effects of the guide vane and draft tube are known, the highest runner efficiency possible cannot be obtained by knowing the proper blade shape at a given layer or section.

In section 9 (b), a crescent shape of blade, as illustrated in Fig. 20, undoubtedly has distinct advantages theoretically, in maintaining a low value of ΔP maximum. For the central portion of the blade, that is, between the periphery and the hub, this shape, for the reasons set forth by the author, might well be used as a guide. At the periphery, however, the blade shape is virtually straight, that is, a/l will be very small, and R_1 will approach a straight line. Here, if the low-pressure side of the blade is formed by a radius R_2, either the thickness t will be unduly great (resulting in reduced lift) or else the blade at each end

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will be too thin for strength and construction. Therefore, for this portion of the blade a profile approaching the airfoil appears reasonable. Near the hub, on the other hand, the relatively great value of t/l demanded by strength considerations requires that the crescent shape be departed from, and that the maximum t be near the leading edge in order to give proper streamline shape to the trailing edge. This again results in a shape approaching the airfoil.

In Table 1 it is noted that there is no tendency for the discharge whirl ratio s to become less with reducing values of specific speed. In fact, the value of s is greatest for the lowest specific speed selected. It is the writer's experience with both propeller and Francis type runners that there is a marked reduction, for best efficiency, in this value of s with lower specific speeds, this value becoming zero at a specific speed of about 27, and below this value becoming negative. It is noted that for pumps in the author's table this tendency does exist.

Figs. 22, 23, and 25 are most interesting as forming a rational guide (for confirmation by actual tests) for the selection of the controlling factors in the design of higher specific speed runners, namely, m, which varies inversely with ϕ^2, and s, the whirl factor at the runner discharge. These figures, however, are for one section or layer of the runner only, and the departure of the blade profile at other sections, from the crescent shape, might result in a materially different picture, for the entire runner, from the relations shown.

The author does not bring into his calculations the number of runner blades. This factor of course has a very decided effect on the efficiency secured. With a given value of l/s, the number of blades determines the axial depth of the profile and the rate of change from $v_{\text{tang } p}$ to $v_{\text{tang } s}$. Therefore an added factor dealing with the number of blades would add to the usefulness of these data.

J. D. Scoville[12] and Henri Deglon.[13] The paper presents a number of interesting and useful formulas dealing with the design problems of pumps and turbines. The author has shown how the theory of Euler can be combined with the airfoil theory. The theory of Euler when properly used gives the correct entrance and discharge angles for a runner having an infinitely large number of blades of infinitesimal thickness. It does not give the proper shape of blade for the practical runner, which must have a few blades of considerable thickness. As the author states, the blades on the practical runner must have somewhat greater curvature than the theoretical runner. It is possible that by means of the two theories the correct angles and shapes may be found to give the maximum efficiency with the minimum danger of cavitation.

For a turbine, $H_1 - H_2$ in Equation [5] is the actual head working on the runner, from which must be subtracted the head lost in friction. In Equations [5] to [7] it is evident that it is the author's intention to introduce the friction, but they are not entirely clear. The friction factor in [6] should be the loss in the runner only and not H_f, which is the sum of H_{fs}, H_{fr}, and H_{fp}.

In combining this with the airfoil theory, the author states that it is customary to neglect the effect of drag in computing the component, since it is only 1 or 2 per cent of the lift. Is this justifiable? The tangential component of the drag is much larger than 1 or 2 per cent of the tangential component of the lift, and to neglect it may introduce an appreciable error. The error is large, especially at the periphery, where the blade angle is small.

The author states that in an ideal fluid the lift is the only reaction acting upon the wing or blade. All forces can be resolved into components parallel and perpendicular to the chord of the wing. For the drag to be zero, the parallel components must balance. Can this be true?

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[13] Engineer, S. Morgan Smith Company, York, Pa.

The writer agrees that the knowledge as to the correct distribution of velocity within the runner is rather meager and that here is a field for considerable research. This lack of knowledge is a weak point in all turbine theories. How does the author arrive at the curve of theoretical velocity distribution shown on Fig. 13?

From formulas [25], [29], and [30] the author designs the blades on the assumption that there are an infinite number of infinitesimal thickness. He then shows in Fig. 16 that when the blades are few in number they must have more curvature than the theory would indicate. How does the author estimate the correction in curvature?

In Fig. 20 the author compares the pressure distribution of the crescent shape of blade with that of an actual airfoil having the same lift and states that the superiority of the crescent is readily appreciated. How do the lift-to-drag ratios of these two forms compare? What are lift coefficients of the two sections?

Author's Closure

(Written at Karlsruhe in December, 1933.)

In order to answer the different writers, the following remarks may be permitted:

Mr. Pfau's statement that "the more the rear surface of the blade conforms to the conditions of a perfect diffuser, the more perfect will be the hydrodynamic suction effect," does not, in the author's opinion, meet the real conditions of turbines. In each turbine, even in a propeller or Kaplan turbine, the relative flow between two adjacent blades is accelerated, and therefore may not be compared with a flow through a diffuser. Furthermore, the formation of surfaces of discontinuity should be carefully avoided, which undoubtedly is possible by providing shockless entrance and by proper shape of leading and trailing edges. The author's developments presume these conditions to be fulfilled, so that the lowest pressure is caused by the unavoidable deflecting effect, which is conducive to an unavoidable differential blade pressure.

Generally it must be realized that danger of cavitation also exists for turbines of low specific speeds when being used for high heads. These turbines simply have a smaller critical value of Σ.

Mr. Pagon mentions the theory of discontinuous flow. Unfortunately, the author is not quite sure what he really means. On the other hand, the author is quite aware that the ideal flow he pictures for carrying out his calculations is not exactly existing. But experience strengthens his opinion that the ideal flow is a very good background for fundamental considerations and that turbulence plays very often only the part of a factor of correction—and that the more, the higher is the Reynolds number.

The induced drag, so important for the theory of airplanes, comes into consideration with respect to turbines—provided an ideal flow is assumed—only when the flow through the gaps between the blades and the curb ring and the hub is discussed, or when conditions of partial load are to be studied, or when it is not possible to arrange the guide vanes for generating a uniform whirl moment.

Mr. Sharp's comments deal with a number of uncertainties that the author himself has pointed out. Especially he feels that the developments of the author, when carried out for the entire runner, not only for the outer layer, would result in a different picture. The author quite agrees, but unfortunately has had so far no time to develop the methods in question for a whole runner. In one of the next volumes of the transactions of his Institute at Karlsruhe (Mitteil. Institut) he expects to be able to do so.

In using well-known airfoil profiles great care must be taken to avoid local cavitation. Profiles advantageously applied to airfoil design often show comparatively thick forms of the leading edges, which, in the flow of a compressible medium, do not do any harm so long as the local surplus of velocity does not approach the velocity of sound, whereas in an incompressible fluid they very soon may create local cavitation. The author prefers forms of leading noses like hatchets with edges rounded only a little. (See reference E1.)

The number of blades is not determined by the author's calculations. Only the ratio l/s comes into consideration. In other words, the theory does not answer the question whether a fixed ratio l/s should be secured by a smaller number of longer blades or a greater number of shorter ones and which way is more favorable for high efficiency. The answer can be given so far only by experience, but the problem is essentially influenced by the conditions of the mechanical design.

Messrs. Scoville and Deglon do not agree with the author's formula [5]. In fact, however, this holds true. The term

$$(v_{\text{tang}\,2}\, u_2 - v_{\text{tang}\,1}\, u_1)/g$$

is equivalent to the work delivered by the shaft to the runner of a pump and, vice versa, by the runner to the shaft of a turbine. Therefore in case of a pump just this term must cover *all* losses except side friction of the runner, leakage, and mechanical losses, whereas in case of a turbine a gross head must be available to cover the above term and *all* losses except the aforementioned.

In case of turbines the terms

$$v_{\text{tang}\,2}\, u_2 - v_{\text{tang}\,1}\, u_1 \quad \text{and} \quad H_2 - H_1$$

become negative; so Equation [5] may be rewritten as:

$$H_1 - H_2 = (v_{\text{tang}\,1}\, u_1 - v_{\text{tang}\,2}\, u_2)/g + H_f$$

which is in accordance with the above.

Neglecting the drag in computing the tangential component is possibly more the author's own than a general custom. But if Messrs. Scoville and Deglon will calculate a very extreme case (for instance $m = 0.15$, $s = 0$, $k = 0.45$), they will find that the effect of drag upon the tangential component is actually of the order of only a few per cent.

In an ideal flow there is no drag at all, which will be understood by recalling that drag is a component not parallel to the chord of the profile, but to the oncoming velocity in case of a single wing and to the mean velocity in case of a series of blades. It is in this way that it is introduced into the author's formulas.

The diagram of Fig. 13 gives only a plot of the whirl moment corresponding to the desired and assumed plot of the differential blade pressure. These two diagrams (plotted against distance x) are entirely conformable to those of Fig. 16. How to form the blades for securing those distributions is explained in connection with Fig. 16.

The distribution of velocity over the spacing between two adjacent blades can be computed theoretically so far only in a few cases by higher mathematical methods. Also the correction in curvature can be exactly determined only in a few simplified cases. The author, in Fig. 16, has estimated it by roughly comparing the effect of a single airfoil profile under similar conditions.

As to Fig. 20, it may be stated that with respect to drag the two profiles can be assumed to differ very little. A fair value of the ratio of drag to lift would be 0.015.

The author wishes to express his sincerest thanks to the A.S.M.E. for the highly appreciated opportunity of discussing vital problems of hydraulic design with American engineers. Also he thanks cordially the engineers for their interest in his paper.